

LMI-BASED STATE-FEEDBACK CONTROL DESIGN FOR A WIND GENERATING POWER PLANT

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Abstract: Several state-feedback design techniques are applied to a multivariable blade pitch controller for a wind turbine generating system comprising a synchronous generator connected to a large power system. Namely; - Ackerman for state feedback and pole placement technique (ACK) - robust LMI-based mixed H_2/H_∞ /Pole placement (ROB) - Static gain Iterative Linear Matrix Inequality (ILMI) controller, design techniques. The proportional part is represented by the system state-feedback while the integral one represents the active control part representing the controlled electrical torque delivered to the power system. To demonstrate the effectiveness of the proposed techniques, divers tests, namely, step/tracking in the controlled variable, variation in system parameters, and a wind speed variation (gust) are applied and the results are discussed.

Key words: Robust Control, ILMI, renewable energy resources, wind generating plant, synchronous generator.

1. Introduction

Wind energy, a form of renewable source, uses wind turbine to convert the energy contained in flowing air into electricity. The main advantages of electricity generation from renewable sources are the absence of harmful emissions and presumed infinite availability of the prime mover that is converted into electricity [1].

Unfortunately, the contribution of wind power is still limited and covers only a small part of the total power system load. The rest is still fulfilled by conventional plants such as thermal, nuclear and hydro power plants. Therefore, the former hardly contributes in voltage and frequency control that are controlled principally by large power plants usually of conventional types. Besides, if a disturbance occurs, the wind turbines are disconnected and reconnected when normal operation has been resumed. The tendency to increase their size will have more influence on overall power system behavior. Thus, their behavior in an electrical power system and interaction with other generating equipments and loads should be looked upon.

For the wind turbines, generated power depends on the wind speed however, for large rotor generator, rotor speed changes are quite smooth. The wind

models describe the fluctuations in the wind speed, which influence the power quality and control characteristics of the wind farm [2-4].

A wind turbine is essentially a machine that converts the kinetic energy of the moving air (wind) first into mechanical energy at the turbine shaft and then into electrical energy. The interaction of the turbine with the wind is complex but a reasonably simple representation is possible. The force of the wind creates aerodynamic lift and drag forces on the rotor blades, which in turn produce the torque on the wind turbine rotor. Wind turbines are designed to produce as much electrical energy as possible. In case of stronger winds it is necessary to waste part of the excess energy of the wind in order to avoid damaging the wind turbine. All wind turbines are therefore designed with some sort of power control. In fact, the wind acting on the rotor plane of a wind turbine is very complex and includes both deterministic effects (mean wind, tower shadow) and stochastic variations due to turbulence [5].

Previous work [6-8] has explored system performance with divers control techniques. In this work, several state-feedback with integral controllers are designed and applied to a large wind turbine power system.

Many control problems and design specifications are expressed as Linear Matrix Inequalities (LMI) that are characterized by [9-11]:

- Able to express a variety of design specifications and constraints.
- Its ability to solve the formulated problem *exactly* by efficient convex optimization algorithms
- Remain tractable for problems where no analytical solution exists

This paper presents the design steps for several proportional-integral state-feedback controllers, namely, Ackerman (ACK) [12], robust LMI-based mixed H_2/H_∞ /pole placement (ROB), static gain iterative LMI (ILMI) [13]. The integral part of the controller is used to achieve zero steady state error in the system output torque. A large wind turbine

driving a synchronous generator connected to a large power system is used [6]. To test the effectiveness of the controllers, divers tests, namely, step/tracking in the controlled variable, variation in system parameters, and wind speed variation (gust) were carried out using Matlab platform with its control and LMI toolboxes.

1. System modeling

The wind turbine converts the kinetic energy of the moving air (wind) first into mechanical energy at the turbine shaft and then into electrical energy as depicted in Fig. 1. In the wind turbine model shown in Fig. 2, the wind speed u together with the blade pitch angle θ_{blade} and rotor speed ω_{rot} are input to the aerodynamic block. The output of the later is the aerodynamic torque, which is the input for the transmission system together with the generator speed. The transmission system has as output the mechanical torque on the high-speed shaft, which is used as an input to the generator model [6,14]. Finally, the blade angle control block models the active control loop, based on the measured torque T_e and the set point T_{ref} .

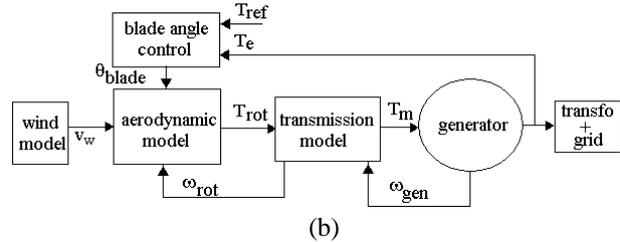


Fig. 2. Wind turbine model

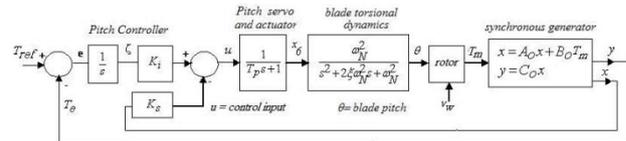


Fig. 3. Block diagram of the system under study

The block diagram of the system under study is shown in Fig. 3. all state variables of the power system are fed feedback and the data is given in the appendix. The state-space representation of the open-loop system, where the variables shown represent small displacements around a selected operating point, can be written as:

$$\dot{x} = Ax + Bu$$

$$\zeta = T_{ref} - Cx$$

$$u = K_i \zeta - K_s x$$

Where

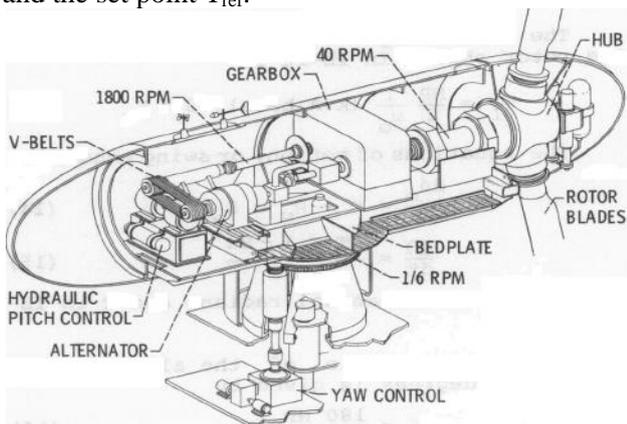
$$x = \begin{bmatrix} \delta & \omega & e'_q & x_4 & x_5 & x_6 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{\omega_o k_1}{2h} & 0 & -\frac{\omega_o k_2}{2h} & \frac{\omega_o k_{th}}{2h} & 0 & 0 \\ \frac{k_4}{\tau_{do}} & 0 & -\frac{1}{\tau_{do}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & \omega_n^2 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_p} \end{bmatrix}$$

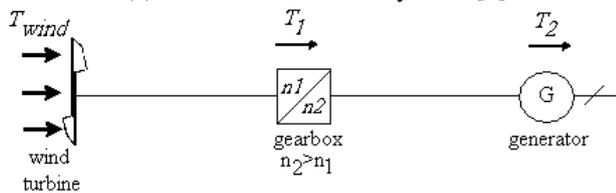
$$C = [k_1 \quad 0 \quad k_2 \quad 0 \quad 0 \quad 0]$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau_p} \end{bmatrix}^t$$

The open-loop system is given by

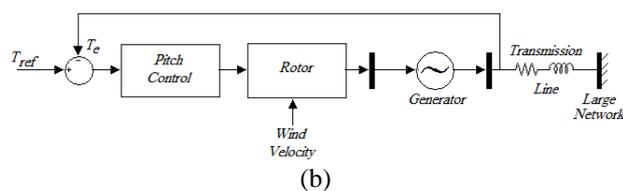


(a) Wind turbine assembly from [1]



(b) Power conversion

Fig. 1. Block diagram of the wind turbine



(b)

$$P_o(s): \begin{cases} \dot{x} = Ax + Bu \\ u = K_i \xi - K_s x \\ \dot{\xi} = T_{ref} - Cx \end{cases} \quad (1)$$

The closed-loop system is given by

$$P_c(s): \begin{cases} \dot{z} = \bar{A}z + \bar{B}T_{ref} \\ u = Kz \\ y = \bar{C}z + \bar{D}T_{ref} \end{cases} \quad (2)$$

Where

$$z = [x \quad \xi]^t$$

$$\bar{A} = \begin{bmatrix} A - BK_s & BK_i \\ -C & 0 \end{bmatrix} \quad \bar{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$K = [-K_s \quad K_i]^t$$

$$\bar{C} = [C \quad 0] = [k_1 \quad 0 \quad k_2 \quad 0 \quad 0 \quad 0 \quad 0]$$

$$\bar{D} = 0$$

The block diagram of the closed-loop system is depicted in Fig. 4.

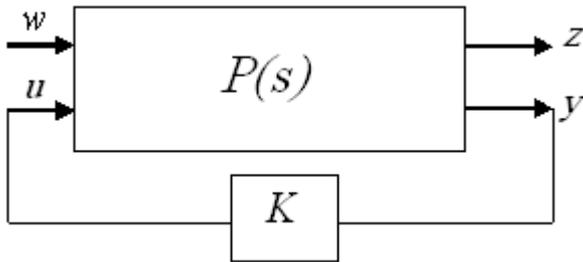


Fig. 4. Closed-loop system

Where

$$w = T_{ref}$$

In the following, the feedback gain matrix is denominated as follows:

- Ackerman: $K = K_{ack}$
- Robust control: $K = K_{rob}$
- Iterative static LMI: $K = K_{ilmi}$

2. Ackerman technique

The integral Ackerman technique for pole placement is used to determine, for controllable system, the feedback gain matrix K_{ack} subject to the placement of the closed-loop eigenvalues is specific locations [12]. The routine "acker" from "matlab" is used as follows:

$$K_{ack} = \text{acker}(\bar{A}, \bar{B}, \lambda_{des})$$

3. H_∞/H_2 LMI robust state-feedback controller

Fig. 5 shows the standard representation of the robust State-Feedback (SF) control block diagram. $P(s)$ represents the plant whereas $K(s) = K_{rob}$ represents the required controller state-feedback gain vector.

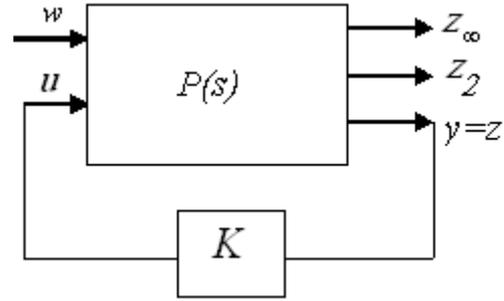


Fig. 5. Standard representation of a robust SF controller

Plant:

$$P(s): \begin{cases} \dot{z} = \bar{A}z + \bar{B}_w w + \bar{B}_u u \\ z_\infty = \bar{C}_1 z + D_{11} w + D_{12} u \\ z_2 = \bar{C}_2 z + D_{21} w + D_{22} u \end{cases} \quad (3)$$

Controller $u = Kz = K_{rob}z$

The H_∞ and H_2 outputs are chosen as follows:

$$z_\infty = e = T_{ref} - T_e = w - \bar{C}z$$

$$z_2 = u$$

Controller:

$$K(s): K_{rob} = [K_i \quad -K_s]$$

(4)

Where

$$\bar{C}_1 = -\bar{C}$$

$$\bar{C}_2 = 0_{1 \times 7}$$

$$D = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\bar{B}_u = \bar{B}$$

$$\bar{B}_w = [B \quad 0]^t$$

The closed-loop system is:

$$\begin{cases} \dot{z} = A_{CL}z + B_{CL}w \\ z_\infty = C_{CL\infty}z + D_{CL\infty}w \\ z_2 = C_{CL2}z + D_{CL2}w \end{cases} \quad (5)$$

with

$$\begin{aligned} A_{CL} &= \bar{A} + \bar{B}_u K & B_{CL} &= \bar{B}_w \\ C_{CL\infty} &= \bar{C}_1 + D_{12} K & C_{CL2} &= \bar{C}_2 + D_{22} K \\ D_{CL\infty} &= D_{11} & D_{CL2} &= D_{21} \end{aligned}$$

The design objectives for finding K_{ROB} are to optimize a mixed objective index, $T = \alpha H_\infty + \beta H_2$, subject to the constraints:

- $H_\infty = \|G_{z,w}(s)\|_\infty < \gamma$
- $H_2 = \|G_{z,w}(s)\|_2 < \nu$
- Place the closed-loop poles in a specific region [12,14].

The closed-loop transfer function is:

$$G_{z,w}(s) = C_{CLi}(s - A_{CL})^{-1} B_{CL} + D_{CLi} \quad i = \infty, 2 \quad (6)$$

The solution is found efficiently using LMI technique. H_∞ -norm represents the system disturbance rejection, i.e., minimization of the effect of the worst-case disturbance w on the output z_∞ . H_2 -norm is used to improve the system performance on the control input u . LMI matlab toolbox routine "msynf" is used.

4. Iterative LMI state-feedback controller

For the linear time-invariant system:

$$\begin{aligned} \dot{z} &= \bar{A}z + \bar{B}u \\ u &= Kz \\ y &= \bar{C}z \end{aligned} \quad (7)$$

The problem is to find a Static State-Feedback (SSF) gain vector $K=K_{ilmi}$ to achieve system dominant eigenvalue to being to the left of a value α in the left half-plane of the s-domain. The algorithm used is described in [13] and is:

Step 0:

Initial data: System's state space realization (A, B, C) then compute $\bar{A}, \bar{B}, \bar{C}$.

Step 1:

Choose $Q_0 > 0$ and solve P for the Riccati equation

$$\bar{A}^T P + P \bar{A} - P \bar{B} \bar{B}^T P + Q_0 = 0, \quad P > 0$$

Set $i=1$ and $X_i = P$.

Step 2:

Solve the following optimization problem for P_i, \bar{F} and α_i .

OP1:

Minimize α_i subject to the following LMI

constraints

$$\begin{bmatrix} \sum_{li} & (\bar{B}^T P_i + \bar{F} \bar{C})^T \\ \bar{B}^T P_i + \bar{F} \bar{C} & -I \end{bmatrix} < 0, \quad P_i > 0 \quad (8)$$

Where

$$\sum_{li} = \bar{A}^T P_i + P_i \bar{A} - X_i \bar{B} \bar{B}^T P_i - P_i \bar{B} \bar{B}^T X_i + X_i \bar{B} \bar{B}^T X_i - \alpha_i P_i$$

Denote by α_i^* the minimized value of α_i .

Step 3:

If $\alpha_i^* \leq 0$, the matrix pair (P_i, \bar{F}) solves SOF problem. Stop. Otherwise go to Step 4.

Step 4:

Solve the following optimization problem for P_i and \bar{F} .

OP2:

Minimize $tr(P_i)$ subject to LMI constraints (8) with $\alpha_i = \alpha_i^*$, where tr stands for the trace of a square matrix. It is equal to the sum of its diagonal elements and also the sum of its eigenvalues. Denote by P_i^* the optimal P_i .

Step 5:

If $\|X_i \bar{B} - P_i \bar{B}\| < \varepsilon$, where ε is a prescribed tolerance. go to Step 6; otherwise set $i := i + 1$, $X_i = P_i^*$, and go to Step 2.

Step 6:

It cannot be decided by this algorithm whether SOF problem is solvable. Stop.

5. Simulation results

To demonstrate the effectiveness of the proposed controllers, several tests are carried out and the results are presented for the wind power plant described earlier being driven by each of the proposed controllers. The simulation results are obtained using MATLAB package, control and LMI Toolboxes.

The system closed-loop eigenvalues are shown in Table 1 whereas the state-feedback gain vectors are shown in Table 2. Important parameters for robust and ILMI controllers are as follows:

ACK:

The desired eigenvalues are shown in Table 1

ROB:

Dominant eigenvalue: $\lambda_d = -1.59$ (desired)

Weight factors: $\gamma_{opt} = 1.3065$, $\nu_{opt} = 0.099877$

ILMI:

The positive definite starting matrix : $Q=10$,
 $\alpha < 0$

The obtained value of α is: -3.1183

Table 1
 Closed-loop eigenvalues

Eigenvalues		
ack	rob	ilmi
-2.0518 ± 100.03i	-373.21	-23848
-15.21 ± 15.282i	-23.724 ± 1.7799i	-499
-4.7586	-5.6145 ± 10.627i	-9.0695 ± 11.017i
-3	-1.6075	-1.5931 ± 1.2939i
-1.59	-4.7423	-6.0269

Table 2
 Closed-loop state-feedback gains

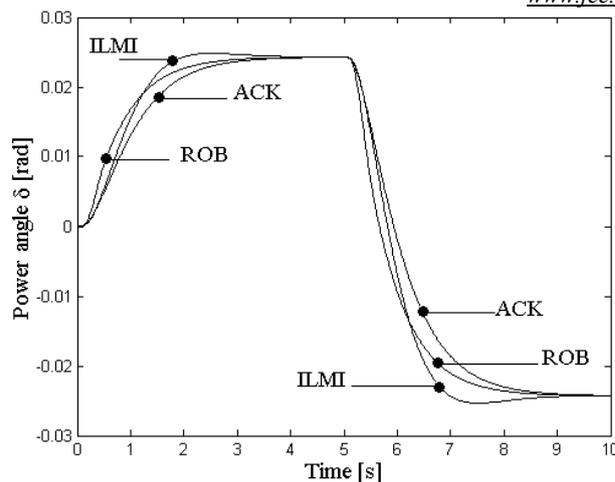
Gains	Acker	Rob	ILMI
K_s	0.11362	0.50069	-28.802
	0.15795	0.12956	79.258
	-0.40701	-0.13535	-203.58
	23.354	0.85724	102.31
	0.85831	0.00211	16.612
K_i	-22.358	0.88771	1411.8
K_i	0.1244	0.27395	72.447

In the following first three tests, the wind speed is held to its nominal value.

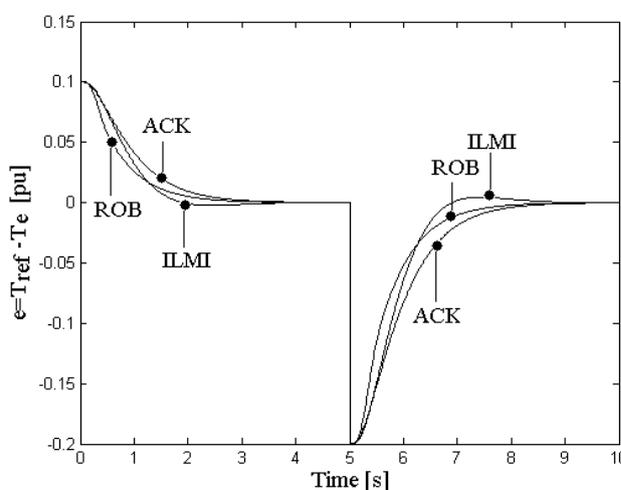
Test 1: Step Response (regulation)

To test the effectiveness of the system equipped with the proposed controllers, the system is subjected to an increase by 1% then a decrease by 1% in T_{ref} (regulation). The time response of the power angle δ , torque error e , electromagnetic torque T_e , control input u , and generator speed ω , are shown in Fig. 6.

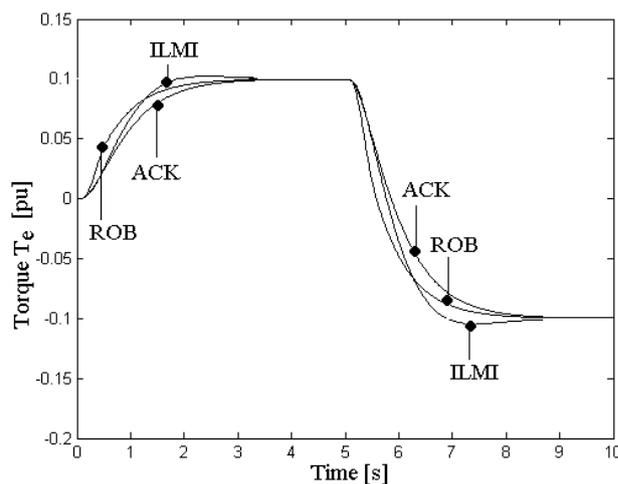
For these specific choices of parameters, All controllers show good responses. However, ILMI is faster and for the generator speed ω , higher overshoots/undershoots are shown for ROB and ILMI during the sudden change in the controlled value.



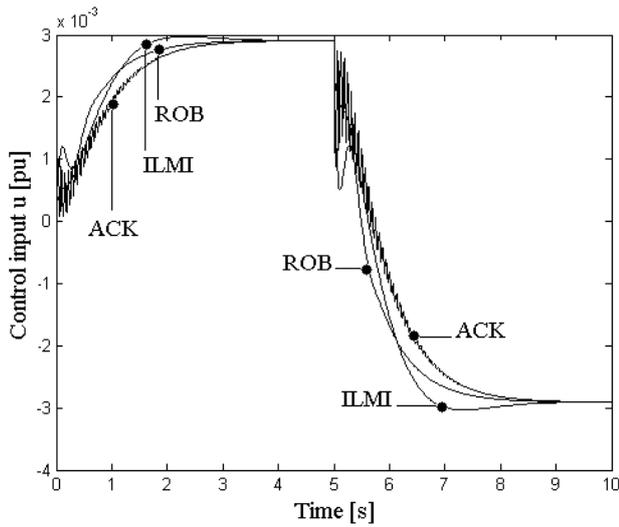
(a) Power angle δ



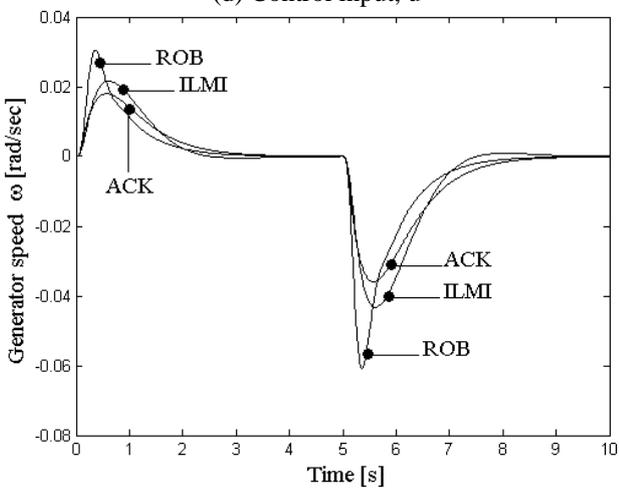
(b) Torque error, e



(c) Electromagnetic torque, T_e



(d) Control input, u

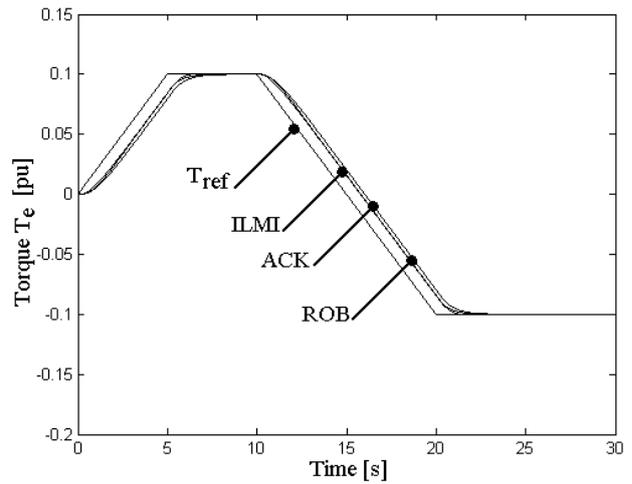


(e) Generator speed ω

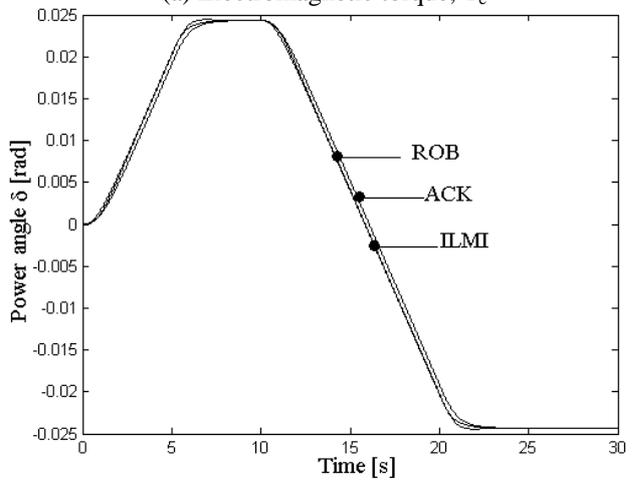
Fig. 6. Step-response following $T_{ref}=+5\%$ the -5%

Test 2: Tracking Response

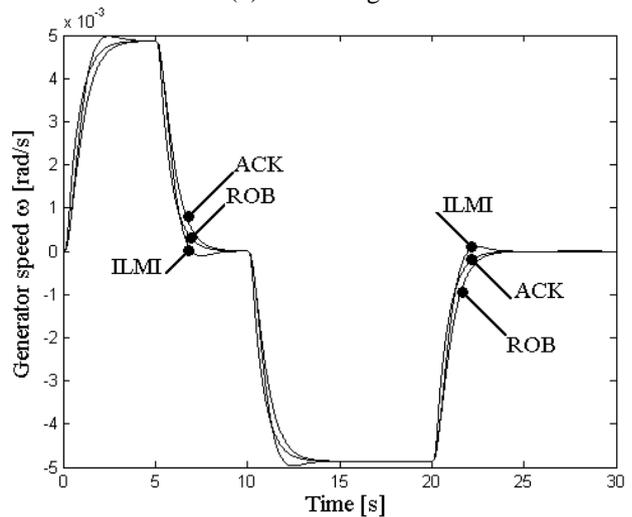
To test the effectiveness of the system to tracking the reference value of the torque, the system is subjected to a variation of T_{ref} as shown in Fig. 7(a). The system response for the electromagnetic torque T_e , power angle δ , and generator speed ω , are shown in Fig. 7(a), Fig. 7(b) and Fig. 7(c), respectively.



(a) Electromagnetic torque, T_e



(b) Power angle δ

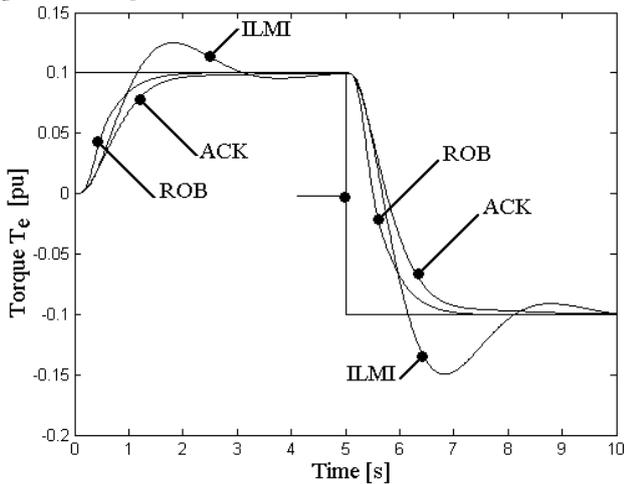


(c) Generator speed ω

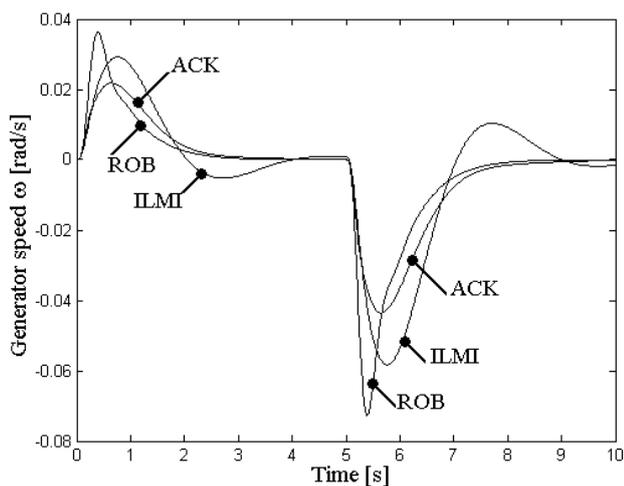
Fig. 7. Tracking the reference torque T_{ref}

Test 3: Parameters Variation

To test the robustness to parameters change, an increase by 50% in the field transient time constant τ_{do} , inertia constant h , developed torque gain k_{th} , machine parameters k_2 and k_3 , time constant of servo-actuator τ_p . Fig. 8 shows the system response following a change in T_{ref} by +1% then by -1% while experiencing the described parameters change. It is worth noting that the state-feedback gain K values used is the one found with nominal system parameters. It is clear that the system responds smoothly with more pronounced overshoots and undershoots for ILMI and ROB especially for the generator speed ω .



(a) Electromagnetic torque, T_e

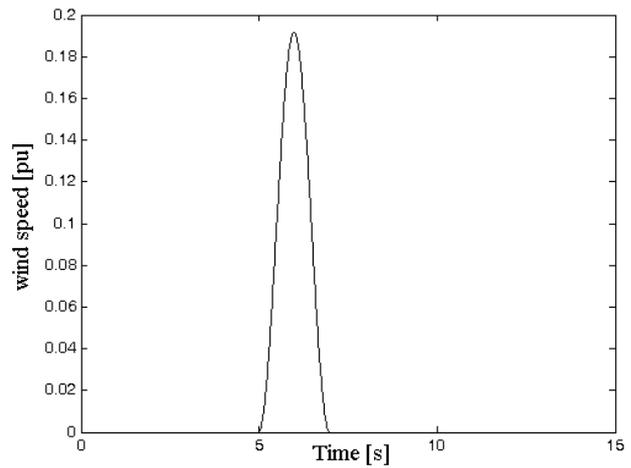


(b) Generator speed ω

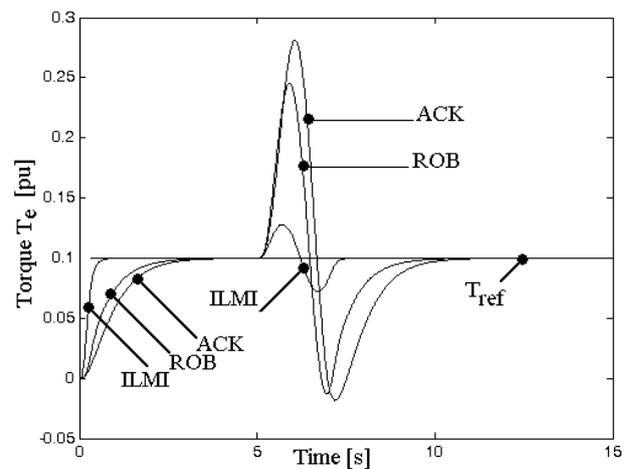
Fig. 8. Increase in system parameters

Test 4: Wind Speed Variation (Gust) effect

Wind gust is a sudden, brief increase in speed of the wind. To test the robustness to the wind speed variation, a gust shown in Fig. 9(a) is applied at $t=5$ seconds. Fig. 9(b) shows the electrical torque behavior following a change in T_{ref} by +10% from $t=0$ second while the gust is applied at 5 seconds [6]. The desired dominant eigenvalues were selected as: $\lambda_{ROB} = -2$, $\lambda_{ACK} = -1.59$, $\lambda_{ILMI} = -5$. It is clear that, for this specific condition, ILMI shows more robustness than ROB and finally than ACK.



(a) Wind speed variation (gust)



(b) Generator torque T_e

Fig. 9. System response due to the presence of a gust

6. Conclusion

Three proportional-integral state-feedback controllers, Ackerman with pole-placement technique (ACK), mixed robust-LMI (ROB), and static-gain iterative-LMI (ILMI), were designed for a power system comprising a wind turbine driving a synchronous generator and connected to an infinite bus via a step-up transformer and a transmission line.

The advantages of one method with respect to

another depends on the designer needs and constraints. Ackerman's method is limited to system pole placement and suffers from system order increase. Further extension of this study will include H_∞ and H_2 to ILMI thus improving its robustness to disturbance variations via H_∞ and its performance via H_2 minimization. Besides, some of the system states are not measurable so observers are needed in this case. The system design is for one operating point, it is worth looking into adaptation of the controllers to different operating points. A switching between the proposed techniques can be done. Where the best choice following a specific disturbance can be selected.

7. Appendix

Table 3
Operating conditions

infinite bus voltage	$V_\infty=1$
reactive power	$Q=0.6$
active power	$P=0.8$
gear ratio	$N=37.5$
Torque factor:	$k_{th}=11.86$
System constants:	$k_1=2.49, k_2=2.51$ $k_3=0.08, k_4=5.14$

Table 4
System data

transmission line resistance	$r_e=0$
transmission line reactance	$x_e=0.02$
turbine speed rpm	$n_t=40$
blade radius	$r_b=62.5$
wind speed m/sec	$v_p=18$
no. of poles	$pp=4$
inertia constant	$h=9.5$
integral gain	$k_p=0.075$
zeta	$\zeta=0.02$
exciter time constant	$\tau=0.05$
exciter gain	$k_e=30$
P.F controller gain	$k_{pf}=0.2$
generator armature resistance	$r_a=0.018$
d-axis reactance	$x_d=2.21$
q-axis reactance	$x_q=1.064$
transient d-axis reactance	$x_d'=0.165$
subtransient d-axis reactance	$x_d''=0.128$
subtransient q-axis reactance	$x_q''=0.193$
d-axis transient field time constant	$\tau_{do}'=1.94212$
d-axis subtransient field time constant	$\tau_{do}''=0.01096$

q-axis subtransient field time constant	$\tau_{qo}''=0.0623$
angular speed of the generator [rad/sec] ω_n	$\omega_o=100\pi$
wind turbine filter time constant	$\tau_p=1/(2*2.7*\pi)$
Torque factor	$k_{th}=11.86$

References

- Freris, L.L.: *Wind Energy Conversion Systems*. Prentice Hall International, 1990.
- Mihet-Popa, L., Blaabjerg, F., Boldea, I.: *Wind Turbine Generator Modeling and Simulation Where Rotational Speed is the Controlled Variable*. In: *IEEE trans. On Ind. Appl.*, Vol. 40, No. 1, Jan/Feb 2004, pp. 3-10.
- Slootweg, J.G., Kling, W.L.: *Modeling of Large Wind Farms in Power System Simulations*. In: *Power Engineering Society Summer Meeting, 2002 IEEE*, Vol. 1, Issue, 25-25 July 2002, pp. 503-508.
- Anca, A.D., Hansen, D., Poul Sorensen, P., Janosi, L.L., Bech, J.: *Wind farm modeling for power quality*. In: *Proc. IEEE IECON'01*, Vol. 3, 2001, pp. 1959-1964.
- Wasynczuk, O., et al.: *Dynamic behavior of a class of wind turbine generators during random wind fluctuations*. In: *IEEE PES Winter Meeting, Atlanta, GA, 1981*.
- Hwang, H., J. Gilbert, L.G.: *Synchronization of wind turbine generators against an infinite bus under gusting wind condition*. In: *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-97, No. 2, March/April 1978, pp. 536-544.
- Javid, S.H., Winkelman, A.M., Winkelman, J.R.: *Control design for a wind turbine-generator using output feedback*. In: *Control Systems Magazine, IEEE* Vol. 2, Issue 3, Sep 1982, pp. 23 - 29.
- Iqbal, M.T., Coonick, A.H., Freris, L.L.: *Some control aspects of a wind turbine*. In: *IEE Paper*, 1993, pp. 6/1-6/3.
- Boyd, S.L., El Ghaoui, E.F., Balakrishnan V.: *Linear Matrix Inequalities in Systems and Control Theory*. SIAM books, Philadelphia, 1994.
- Gahinet, P., Apkarian, P.: *A Linear Matrix Inequality Approach to H_∞ Control*. In: *Int. J. Robust and Nonlinear Control*, 4 (1994), pp. 421-448.
- Gahinet, P., Nemirovski, A., Laub, A.J., Chilali, M.: *LMI Control Toolbox for Use with MATLAB*. The MathWorks User's Guide, Version 1, May 1995.
- Ogata, K.: *Modern control engineering*. Prentice Hall, 3rd Ed., NJ, 1997.
- Cao, Y.Y., Lam, J., Sun, Y.X.: *Static Output Feedback Stabilization: an ILMI approach*. In: *Automatica* 34 (1998), pp. 1641-1645.
- Anderson, P.M., Fouad, A.A.: *Power System Control and Stability*. IEEE Press, 1993.