Performance Analysis and Comparative Study of LMI-Based Iterative PID Load-Frequency Controllers of a Single-Area Power System

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Abstract: - This paper provides a comparison of the performance and the design steps for four robust static output feedback Proportional-Integral-Derivative (PID) controllers. The first one presents an Iterative PID (IPID) that guarantees the stability of the closed-loop system. In the second and the third controllers, Iterative PID based on H_2 (IPIDH2) and H_∞ (IPIDHI) performances, respectively, are investigated. The role of H_∞ is to minimize the disturbance effect whereas H_2 is used to improve the transients of the system output. The last one is an Iterative PID that is characterized by the Maximum (IPIDM) regulated output of the closed loop system with its command input being bounded. The Iterative Linear Matrix Inequality (ILMI) technique is developed to find the feedback gains of the designed PID controllers. The proposed design technique is applied to the Load Frequency Control (LFC) problem of a single-area power system. The effects of ILMI tuning variables on the system dynamic response are given and discussed. To test and compare the effectiveness of the controllers, diverse simulation tests are carried out under diverse disturbances and parameters change with the presence of the Generation Rate Constraint (GRC), inherent system nonlinearity. The results prove that the proposed iterative PID controllers are very useful for LFC power system.

Key-Words: - Load frequency control, Iterative PID, Linear matrix inequalities, H-infinity norm, H-2 norm, maximum power output control.

1 Introduction

Even though the wide popularity of the Proportional-Integral-Derivative (PID) controller in the industrial world, its parameters are usually tuned manually or using trial-and-error approach or by conventional control methods. Therefore, it is incapable of obtaining good dynamical performance to capture all design objectives and specifications for a wide range of operating conditions and disturbances [1-3].

In the literature, many design criteria are based on optimization techniques; starting from classical to intelligent ones such as genetic algorithm, evolutionary programming, particle swarm and simulated annealing. They have been applied to the Load Frequency Control (LFC) problem of a power system [4-7].

Iterative Linear Matrix Inequality (ILMI) method, proposed in [8-10] and later employed to design Static Output Feedback (SOF) controller [11-13], represents a new tool that attracted considerable attention and played an important role in control applications [14-16]. It is used to calculate the PID controller gains.

Unlike the state feedback case, SOF gains, which stabilize the system, are not easily found [8-13]. Linear Matrix Inequality (LMI) is one of the most effective and efficient tool in control design [17-19]. The conjunction between LMI and SOF represents a new technique in designing

Iterative PID controller fulfilling some desired constraints such as:

- Guaranty of the system stability (**IPID**),
- Improvement of the system transients response through H_2-specification (**IPIDH2)**
- Reduction of the system disturbance effect through H_∞-specification **(IPIDHI)**
- Limitation of the system output through Maximum Output Control, MOC, **(IPIDM**).

In addition, the iterative PID controllers are provided with tuning variables that have an important influence on the system dynamics that are not usually found in every controller. Moreover, its superiority resides in its simplicity: no need for unmeasurable states or observers, no telemetry problems, and suitability for on-line applications since only measured outputs are used.

Based on the above, this paper provides design steps and presents a comparison between the performances of four robust iterative PID controllers. The first one, IPID, guarantees the closed-loop system stability with possibility of dominant eigenvalue shift to the left hand-side of Laplace plane. The second, IPIDH2, improves the system transient response while the third, IPIDHI, minimizes the disturbance effect on the system output. The fourth one, IPIDM, forces the system output to be less than a specific value for a bounded input signal [9,10]. The proposed controllers are applied to a LFC of a power system comprising a single area. A comparative study of the controlled system driven by the each of the proposed controllers are carried out and the results compared to the system with the presence of the Generation Rate Constraint (GRC), inherent nonlinearity, and wide range of parameters variation [20]. The obtained results are very encouraging to pursue further investigation.

2 Power System Modelling

Figure 1 shows the transfer function model of the LFC for a single area power system controlled by a PID controller.

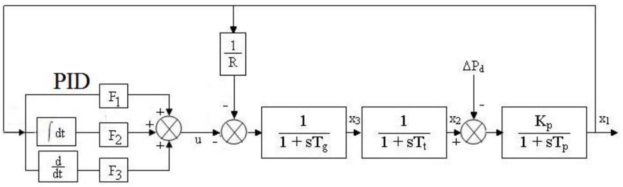


Fig.1 Block Diagram of a single area LFC.

The continuous linear dynamic open-loop model, in state-space form, can be written as [4-7, 21]:

$$\begin{cases} \dot{x} = Ax + Bu + Fd \\ y = Cx + Du \end{cases} \quad (1)$$

Where,

- x state vector (3x1),
- y output (3x1)
- u disturbance and control vector (1x1)
- F_{3x1} disturbance matrix
- A_{3x3}, B_{3x1}, C_{1x3}, D_{1x1} constant matrices
- d disturbance vector (ΔP_d)

In the block diagram illustrated by Fig. 1:

- Tp plant model time constant
- T_t turbine time constant
- T_g governor time constant
- K_p plant gain
- R speed regulation due to governor action
- x_1 change in system frequency
- x_2 incremental changes in generator output
- x_3 governor valve position
- F_1, F2, F_3 gains of the PID controller.

The control objective in the LFC problem is to keep the change in the frequency (ΔF= x1) as close to zero as possible when the system is subjected to a load disturbance (d=ΔPd) by manipulating the controlled input (u). The system matrices have the following form [4-7, 18].

$$A = \begin{bmatrix} -\frac{1}{T_p} & \frac{K_P}{T_P} & 0 \\ 0 & -\frac{1}{T_p} & \frac{1}{T_p} \\ -\frac{1}{RT_g} & 0 & -\frac{1}{T_g} \end{bmatrix} \qquad B^T = \begin{bmatrix} 0 & 0 & \frac{1}{T_g} \end{bmatrix}^T$$

$$F = \begin{bmatrix} \frac{K_P}{T_P} & 0 & 0 \end{bmatrix}^T \qquad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

The system parameters are:

K_p = 120 pu
T_p = 20 s
T_t = 0.3 s
T_g = 0.08 s
R = 2.4 Hz/pu.MW

3 Iterative PID (IPID)

In Fig. 2, a transformation form of the PID to a SOF controller is performed by considering the linear time-invariant system given by (1) and rewritten as follows [9,10]:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad (2)$$

With the following PID controller

$$u = F_1 y + F_2 \int_0^t y dt + F_3 \frac{dy}{dt} \quad (3)$$

Where $x \in R^n$ is the state variables, $u(t) \in R^l$ is the control inputs, $y \in R^m$ is the outputs, A, B and C are matrices with appropriate dimensions, and F_1, F_2 and $F_3 \in R^{l \times m}$ are matrices to be designed (PID gains).

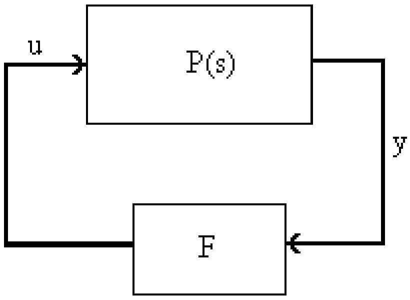


Fig. 2 Multivariable PID reorganized as a SOF controller

Let $\begin{cases} z_1 = x \\ z_2 = \int_0^t y dt \end{cases}$ and denote $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$ the variable that can be viewed as the state vector of a new system whose dynamics are governed by

$$\begin{cases} \dot{z}_1 = \dot{x} = Az_1 + Bu \\ \dot{z}_2 = y = Cz_1 \end{cases} \quad (4)$$

Or, in compact form,

$$\dot{z} = \bar{A}z + \bar{B}u \quad (5)$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \qquad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

Combining (2) and (5) yields

$$y = \begin{bmatrix} C & 0 \end{bmatrix} z, \qquad \int_0^t y dt = \begin{bmatrix} 0 & I \end{bmatrix} z$$

$$\frac{dy}{dt} = \begin{bmatrix} CA & 0 \end{bmatrix} z + CBu .$$

And

$$\bar{y}_i = \bar{C}_i z, \qquad i = 1,2,3$$

$$u = \bar{F}\,\bar{y} = \bar{F}_1 \bar{y}_1 + \bar{F}_2 \bar{y}_2 + \bar{F}_3 \bar{y}_3 \quad (6)$$

$$\bar{y} = \bar{\bar{F}} \bar{y}$$

where

$$\bar{C}_1 = \begin{bmatrix} C & 0 \end{bmatrix}, \bar{C}_2 = \begin{bmatrix} 0 & I \end{bmatrix}, \bar{C}_3 = \begin{bmatrix} CA & 0 \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} \bar{C}_1^T & \bar{C}_2^T & \bar{C}_3^T \end{bmatrix}^T$$

And if $(I - F_3 CB)$ is invertible, then

$$\bar{F}_i = (I - F_3 CB)^{-1} F_{i1} \qquad (i = 1-3) \quad (7)$$

Thus, the problem of a PID controller design reduces to that of a SOF controller design for the following system [9-10]:

$$\begin{cases} \dot{z} = \bar{A}z + \bar{B}u \\ \bar{y} = \bar{C}\,z \\ u = \bar{F}\,\bar{y} \end{cases} \quad (8)$$

Once $\bar{F} = \begin{vmatrix} \bar{F}_1 & \bar{F}_2 & \bar{F}_3 \end{vmatrix}$ is found, the original PID gains can be recovered from

$$F_3 = \bar{F}_3 (I + CB\bar{F}_3)^{-1}$$
$$F_2 = (I - F_3 CB)\bar{F}_2 \quad (9)$$
$$F_1 = (I - F_3 CB)\bar{F}_1$$

3.1 IPID Algorithm:

Step 0: Initial data: System's state space realization (A, B, C) then compute $\bar{A}, \bar{B}, \bar{C}$

Step 1: Choose $Q_0 > 0$ and solve P for the Riccati equation

$$\bar{A}^T P + P\bar{A} - P\bar{B}\bar{B}^T P + Q_0 = O, \quad P > 0$$

Set i= 1 and X_1 =P.

Step 2: Solve the following optimization problem for P_i, $\bar{F}$ and α_i.

OP1: Minimize α_I subject to the following LMI constraints

$$\begin{bmatrix} \Sigma_{1i} & (\bar{B}^T P_i + \bar{F}\bar{C})^T \\ \bar{B}^T P_i + \bar{F}\bar{C} & -I \end{bmatrix} < 0, \quad P_i > 0 \quad (10)$$

Where

$$\Sigma_{1i} = \bar{A}^T P_i + P_i \bar{A} - X_i \bar{B}\bar{B}^T P_i - P_i \bar{B}\bar{B}^T X_i + X_i \bar{B}\bar{B}^T X_i - \alpha_i P_1$$

Denote by α_i* the minimized value of α_i.

Step 3: If $\alpha_i^* \le 0$, the matrix pair (P_i, $\bar{F}$) solves SOF problem. Stop. Otherwise go to Step 4.

Step 4: Solve the following optimization problem for P_i and $\bar{F}$.

OP2: Minimize tr(P_i) subject to LMI constraints (10) with $\alpha_i = \alpha_i^*$, where tr stands for the trace of a square matrix. It is equal to the sum of its diagonal elements and also the sum of its eigenvalues.

Denote by P_i* the optimal P_i.

Step 5: If $\| X_i \bar{B} - P_i \bar{B} \| < \varepsilon$, where ε is a prescribed `tolerance. Go to Step 6; Otherwise, i= i+1, $X_i = P_i^*$, and go to Step 2.

Step 6: It cannot be decided by this algorithm whether SOF problem is solvable. Stop.

The initial data for the IPID algorithm is given by steps 0 and 1 and are only used in the starting. The optimization problem **OP1,** in Step 2, is a *generalized eigenvalue minimization* problem. This step guarantees the progressive reduction of α_i whereas Step 3 guarantees the convergence of the algorithm. If the later fails to arrive to a stabilizing solution, then select another value for Q and execute the algorithm. The optimization problems **OP1** and **OP2** in step 2 and step 4 are performed, respectively, using *gevp* and *mincx* routines in Matlab LMI toolbox [19].

4 Iterative PID with H∞ (IPIDHI)

The design problem of a IPID controller under H_∞-performance specification is presented by considering the system (1), with a disturbance w, written as [9,10]

$$\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ y_s = C_s x \\ y_r = C_r x + Du \end{cases} \tag{11}$$

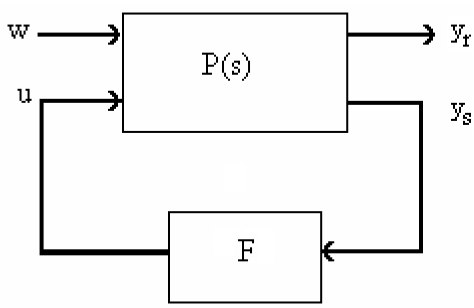


Fig. 3 Closed-loop system via IPIDHI Controller

With the PID controller defined as

$$u = F_1 y_s + F_2 \int_0^t y_s dt + F_3 \frac{dy_s}{dt} \tag{12}$$

As before, x is state variable, w is the disturbance and other external input vector, u is the control inputs, y_r is the controlled output vector and y_s is the measured output vector. A, B_1, B_2, C_r, and C_s are matrices with appropriate dimensions, and F_1, F_2, F_3 are matrices to be designed.

The output feedback H∞-control problem is to find a controller of the form

$$u = Fy_s \tag{13}$$

such that the infinite norm of the closed-loop transfer function from w to y_r is stable and

$$\| T_{wyr} \|_\infty < \gamma \tag{14}$$

Using similar transformation as before, one ends up to the following set of equations where the problem of a PID is reduced to a SOF control system:

$$\begin{cases} \dot{z} = \overline{A}z + \overline{B}_1 w + \overline{B}_2 u \\ \overline{y}_s = \overline{C}_s z \\ \overline{y}_r = \overline{C}_r z + Du \\ u = \overline{F}\,\overline{y}_s \end{cases} \tag{15}$$

Where

$$\overline{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \quad \overline{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \quad \overline{B}_2 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix}$$

$$\overline{C}_s = \begin{bmatrix} C_s & 0 \end{bmatrix} \quad \overline{C}_r = \begin{bmatrix} C_r & 0 \end{bmatrix}$$

$$\overline{F} = \begin{bmatrix} \overline{F}_1 & \overline{F}_2 & \overline{F}_3 \end{bmatrix}$$

Once $\overline{F}$ is found using the following algorithm and assuming that $(I-F_3C_sB_1)$ is invertible then the original PID gains can be recovered from (9).

4.1 IPIDHI Algorithm:

Step 0: Initial data: System's state space realization (A, B_1, B_2, C_s, C_r, D) and performance index γ then compute $\overline{A}, \overline{B}_1, \overline{B}_2, \overline{C}_s, \overline{C}_r$ defined in (15).

Step 1: Choose $Q_0 > 0$ and solve P for the Riccati equation

$$\overline{A}^T P + P\overline{A} - P\overline{B}_2\overline{B}_2^T P + Q_0 = O, \quad P > 0$$

Set i= 1 and $X_1 = P$.

Step 2: Solve the following optimization problem for P_i, $\overline{F}$ and α_i.

OP1: Minimize α_I subject to the following LMI constraints

$$\begin{bmatrix} \Sigma_{3i} & P_i\overline{B}_1 & (\overline{C}_r + D\overline{F}\overline{C}_s)^T & (\overline{B}_2^T P_i + \overline{F}\overline{C}_s)^T \\ \overline{B}_1^T P_i & -\gamma I & 0 & 0 \\ \overline{C}_r + D\overline{F}\overline{C}_s & 0 & -I & 0 \\ \overline{B}_2^T P_i + \overline{F}\overline{C}_s & 0 & 0 & -I \end{bmatrix} < 0 \tag{16}$$

$$P_i > 0$$

Where

$$\Sigma_{3i} = \overline{A}^T P_i + P_i\overline{A} - X_i\overline{B}_2\overline{B}_2^T P_i - P_i\overline{B}_2\overline{B}_2^T X_i + X_i\overline{B}_2\overline{B}_2^T X_i - \alpha_i P_1$$

Denote by α_i* the minimized value of α_i.

Step 3: If $\alpha_i^* \le 0$, the matrix pair (P_i, $\overline{F}$) solves the problem. Stop. Otherwise go to Step 4.

Step 4: Solve the following optimization problem for P_i and $\overline{F}$.

OP2: Minimize $tr(P_i)$ subject to LMI constraints (16) with $\alpha_i = \alpha_i^*$. Denote by P_i^* the optimal P_i.

Step 5: If $\| X_i\overline{B} - P_i^*\overline{B} \| < \varepsilon$. Where ε is a prescribed tolerance, go to Step 6; otherwise set i:= i + 1, $X_i = P_i^*$,
go to Step 2.

Step 6: It cannot be decided by this algorithm whether SOF problem is solvable. Stop.

In short, Steps 0 and 1, from the algorithm, represent the initial data for the algorithm that are used only during the starting. The optimization problem **OP1** in Step 2 is a generalized eigenvalue minimization problem (*gevp* in Matlab LMI toolbox). This step guarantees the progressive reduction of α and leads to a stable system. In addition, minimization of the disturbance effect on the selected outputs is obtained. Step 3 guarantees the convergence of the algorithm. If the algorithm fails to arrive to the stabilizing solution, **OP2** in step 4 is performed to minimize the *trace* of *P* using *mincx* (in Matlab LMI toolbox). In this case, the real value of the dominant eigenvalue is shifted to the left side. If a solution cannot be decided from the iterative process, used another value for Q_0 and possibly γ then rerun the algorithm. If no stabilizing solution is obtained, then it cannot be decided by this algorithm whether PIDHI problem is solvable.

5 Iterative PID with H_2 (IPIDH2)

The design problem of PID controller under H_2 performance specification is investigated, first, by studying the static output feedback (SOF) case and then extending the result to the PID case. As before, consider the system [9,10]:

$$P(s):\begin{cases}\dot{x}=Ax+Bu\\ y=Cx\end{cases} \tag{17}$$

Assuming that A is stable then for the system closed-loop transfer function

$$G(s)=C(sI-A)^{-1}B+D \tag{18}$$

the classical result within Lyapunov approach gives

$$\|G\|_2^2=Trace(CPC^T) \tag{19}$$

where P is a solution of the following Lyapunov equation:

$$AP+PA^T+BB^T=0 \tag{20}$$

The Static Output Feedback with H_2 performance control (SOFH2) problem is to find a control of the form

$$u=Fy_s \tag{21}$$

such that the closed-loop transfer function, from w to y_r, is stable and

$$\|G_{wyr}\|_2<\gamma \tag{22}$$

with $\gamma>0$ and $\|.\|_2$ denotes the *2*-norm of the system transfer matrix.

The H_2-performance index, for system (17) rewritten as

$$P(s):\begin{cases}\dot{x}=Ax+B_1w+B_2u\\ y_s=C_sx\\ y_r=C_rx\end{cases} \tag{23}$$

can be achieved by a SOF controller if the matrix inequalities:

$$\begin{cases}trace\,(C_rPC_r^t)<\gamma^2\\ AP+PA^T-PC_s^TC_sP+(B_2F+PC_s^T)(B_2F+PC_s^T)^T+B_1B_1^T<O\\ P>0\end{cases} \tag{24}$$

have solutions for (P,F).

The PID design with H_2 specifications converts to a SOF control for the dynamics of the newly obtained system:

$$\begin{cases}\dot{z}=\overline{A}z+\overline{B}_1w+\overline{B_2}u\\ \overline{y}=\overline{C}_sz\\ \overline{y}_r=\overline{C}_rz\\ u=\overline{F}\overline{y}_s\end{cases} \tag{25}$$

Where

$$\overline{A}=\begin{bmatrix}A & 0\\ C_s & 0\end{bmatrix}\quad \overline{B}1=\begin{bmatrix}B_1\\ 0\end{bmatrix}\quad \overline{B}2=\begin{bmatrix}B_2\\ 0\end{bmatrix}$$

$$\overline{C}_{s1}=\begin{bmatrix}C_s & 0\end{bmatrix}\quad \overline{C}_{s2}=\begin{bmatrix}0 & I\end{bmatrix}\quad \overline{C}_{s3}=\begin{bmatrix}C_s & 0\end{bmatrix}$$

$$\overline{C}_s=\begin{bmatrix}C_{s1}^T & C_{s2}^T & C_{s3}^T\end{bmatrix}^T\quad \overline{C}_r=\begin{bmatrix}C_r & 0\end{bmatrix}$$

Thus, once the feedback matrices $\overline{F}=(\overline{F}_1,\overline{F}_2,\overline{F}_3)$ are obtained using the following iterative LMI algorithm for solving H_2-SOF control, the original PID gains $F=(F_1,F_2,F_3)$ can be recovered from (9). In the algorithm, use the following:

$$A=\overline{A},B_1=\overline{B}_1,B_2=\overline{B}_2,C_s=\overline{C}_s,C_r=\overline{C}_r,F=\overline{F}$$

5.1 IPIDH2 Algorithm:

Step 0: Form the system state space realization: (A,B_1,B_2,C_s,C_r) and select the performance index γ

Step 1: Choose $Q_0>0$ and solve P for the Riccati equation:

$$AP+PA^T\text{-}PC_S^TC_SP+Q_0=O,\quad P>0$$

Set i= 1 and $X=P$

Step 2: Solve the following optimization problem for P_i, F and α_i.

OP1: Minimize α subject to the following LMI constraints

$$\begin{bmatrix} \Sigma_2 & B_2F + PC_s^T \\ (B_2F + PC_s^T)^T & -I \end{bmatrix} < 0$$

$$trace(C_r PC_r^T) < \gamma^2 \quad (26)$$

$$P > 0$$

Where

$$\Sigma_2 = AP + PA^T + B_1B_1^T - XC_s^T C_s P - PC_s^T C_s X + XC_s^T C_s X - \alpha.P$$

Denote by α^* the minimized value of α.

Step 3: If $\alpha^* \le 0$, the matrix pair (P,F) solves the problem. Stop. Otherwise go to Step 4.

Step 4: Solve the following optimization problem for P and F.

OP2: Minimize $trace(P)$ subject to LMI constraints (26) with $\alpha = \alpha^*$. Denote by P* the optimal P.

Step 5: If $\|XB-P^*B\|<\varepsilon$. where ε is a prescribed tolerance, go to Step 6;
Otherwise set i= i+1, $X=P^*$, go to Step 2.

Step 6: It cannot be decided by this algorithm whether the problem is solvable. Stop.

6 Maximum Power Control (IPIDM)

The design problem of a PID controller under the performance requirement that the system output y_r is smaller than a specified value σ when the input signal w is bounded, is known as Maximum Output Control (MOC) problem. To handle such problem, consider the system [9,10]

$$\begin{cases} \dot{x} = Ax + B_1w + B_2u \\ y_s = C_s x \\ y_r = C_r x + Du \end{cases} \quad (27)$$

With x(0)=0.

The Static Output Feedback Maximum Output Control (SOFMOC) problem is to find a control of the form (22) such that the maximum regulated output $Y_{r,max}$, from w to y_r, of the closed-loop system, under the command input w, satisfies

$$Y_{r,max} = sup_{t \ge 0} \|y_r(t)\| \le \sigma, \ (\sigma > 0) \quad (28)$$

This is fulfilled if there exist matrices P>0 and F, and numbers $\tau_2 \ge 0$, η>0, such that the following linear matrix inequalities hold [9,10]:

$$\begin{bmatrix} P & (C_r + DFC_s)^T \\ (C_r + DFC_s) & \frac{\sigma^2}{\eta} I \end{bmatrix} > 0 \quad (29)$$

$$\begin{bmatrix} \Sigma_3 & PB_1 \\ B_1^T P & -\tau_2 \eta I \end{bmatrix} < 0$$

Where $\Sigma_3 = (A + B_2FC_s)^T P + P(A + B_2FC_s) + \tau_2 P$.

The PID design with MOC specifications converts to a SOFMOC for the dynamics of the newly obtained system

$$\begin{cases} \dot{z} = \overline{A}z + \overline{B}_1 w + \overline{B}_2 u \\ \overline{y} = \overline{C}_s z \\ \overline{y}_r = \overline{C}_r z + Du \\ u = \overline{F}\overline{y}_s \end{cases} \quad (30)$$

So, the following algorithm can be applied to (30) using

$$A = \overline{A}, B_1 = \overline{B}_1, B_2 = \overline{B}_2, C_s = \overline{C}_s, C_r = \overline{C}_r, F = \overline{F}$$

Where

$$\overline{A} = \begin{bmatrix} A & 0 \\ C_s & 0 \end{bmatrix} \quad \overline{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \quad \overline{B}_2 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix}$$

$$\overline{C}_{s1} = \begin{bmatrix} C_s & 0 \end{bmatrix}, \ \overline{C}_{s2} = \begin{bmatrix} 0 & I \end{bmatrix}, \ \overline{C}_{s3} = \begin{bmatrix} C_s & 0 \end{bmatrix}$$

$$\overline{C}_s = \begin{bmatrix} C_{s1}^T & C_{s2}^T & C_{s3}^T \end{bmatrix}^T, \ \overline{C}_r = \begin{bmatrix} C_r & 0 \end{bmatrix}$$

As before, to recover the original PID gains $F = (F_1, F_2, F_3)$ from the feedback matrices $\overline{F} = (\overline{F}_1, \overline{F}_2, \overline{F}_3)$, the relations in (9) can be applied.

An iterative LMI algorithm (*Algorithm 3*) for solving SOFMOC is developed in [9,10] as follows:

6.1 IPIDM Algorithm:

Step 0: Let the system state space realization (A, B_1, B_2, C_s, C_r, D), a performance index σ, and a given number η>0 be given

Step 1: Choose Q_0 > 0 and solve P for the Riccati equation:

$$A^T P + PA - PB_2B_2^T P + Q_0 = O, \quad P > 0$$

Set i= 1 and $X = P$

Step 2: Solve the following optimization problem for P, F and α.

OP1: Minimize α subject to the following LMI constraints

$$\begin{bmatrix} \Sigma_4 & PB_1 & (B_2^T P + FC_s)^T \\ B_1^T P & -\tau_2 \eta I & 0 \\ B_2^T P + FC_s & 0 & -I \end{bmatrix} < 0 \quad (31)$$

$$\begin{bmatrix} P & (C_r + DFC_s)^T \\ (C_r + DFC_s) & \frac{\sigma^2}{\eta} I \end{bmatrix} > 0$$

$$P > 0$$

Where

$$\Sigma_4 = A^T P + PA + XB_2 B_2^T P - PB_2 B_2^T X + XB_2 B_2^T X + \tau_2 P - \alpha P$$

Denote by α* the minimized value of α.

Step 3: If α*≤0, the matrix *F* solves the problem. Stop. Otherwise go to Step 4.

Step 4: Solve the following optimization problem for *P, F*.

OP2: Minimize *trace*(*P*) subject to LMI constraints (31) with α = α*. Denote by P* the optimal P.

Step 5: If ||*XB-P*B*||<ε. where ε is a prescribed tolerance, go to Step 6; otherwise set i= i+1, $X=P^*$,

$$\tau_2 = \sqrt{\frac{trace(P^* B_1 B_1^T P^*)}{\eta\ tr(P^*)}}$$

and go to Step 2.

Step 6: It cannot be decided by this algorithm whether the SOFMOC problem is solvable. Stop.

7 Simulation Results

The simulation results are obtained using the MATLAB package and the LMI Toolbox. Firstly, the simulation is started by illustrating the effect of the tuning variables, used in the algorithms described above, on the dynamic response for the LFC of a single-area power system driven by each of the proposed PID controllers: IPID, IPIDHI , IPIDH2 and IPIDM, as follows:

7.1 Effect of Q on IPID

The tuning variables of the PID designed using iterative LMI are Q, α and of the iteration number *i*. The most affecting parameter is Q (coefficient of positive definite starting matrix). Figure 4 shows the influence of Q on ΔF when the area is subjected to a ΔP_d =+0.5 % (load change) while fixing the other tuning variables. On the basis of the simulation results, it is clearly seen that Q has an important influence on the responses when the system is driven by the controller designed with different values of Q under constant value of *i=50* and α =-0.1. Table 1 illustrates the influence of Q and α on the gains of the IPID controller and the system damping. The system eigenvalues and the damping ratio ζ are also shown in the same Table.

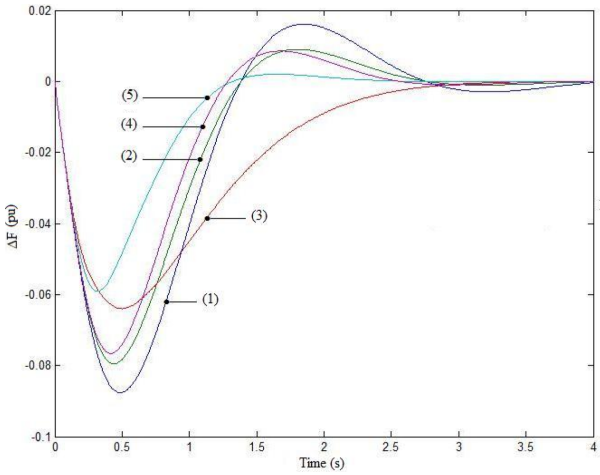


Fig. 4 Effect of different values of Q ($1:Q_1$, $2:Q_2$, $3:Q_3$, $4:Q_4$, $5:Q_5$)

7.2 Effect of γ on IPIDHI

The tuning variables of the PID designed with H_∞-norm using iterative LMI are Q, γ, α and the algorithm convergence (iteration) number. It is noticed that γ has the largest effect. The effect of Q is similar to that mentioned in the IPID. So, Q is fixed while α is varied as shown in Fig. 5. From the figure, it is clearly shown that γ has a great influence on the system frequency ΔF for a step change ΔP_d=0.5 %. The other tuning variables are selected using the following suitable values: Q = 600, i=50 and α =-1.

Table 2 illustrates the influence of γ on the desired α, controller gains, system eigenvalues λ and its damping ratio ζ .

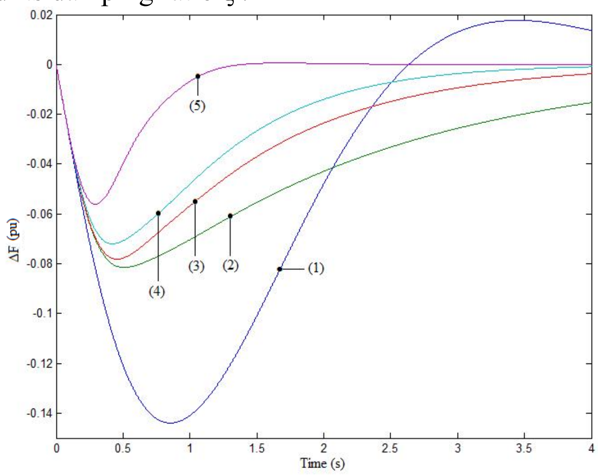


Fig. 5 Effect of different values of γ ($1:\gamma_1$, $2:\gamma_2$, $3:\gamma_3$, $4:\gamma_4$, $5:\gamma_5$)

7.3 Effect of γ on IPIDH2

Similar disturbance is applied to the controlled system with IPIDH2 to illustrate the effect of the tuning variables. Here, the number of iterations i of the algorithm represents an important tuning variable. Besides, γ has an also an important effect on the system dynamic response. Figure 6 shows the system dynamic response with suitable values: Q = 600 and α =-1. It is clear that both γ and i are highly system effective as tuning variables. Table 3 illustrates the effect of these tuning variables on the controlled system dynamics together with system eigenvalues and damping ratio ζ.

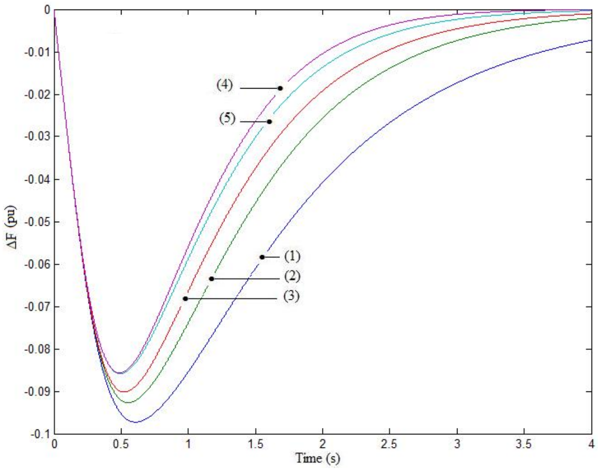


Fig. 6 Effect of different values of γ ($1:\gamma_1$, $2:\gamma_2$, $3:\gamma_3$, $4:\gamma_4$, $5:\gamma_5$)

7.4 Effect of γ and σ on IPIDM

7.4.1 Effect of σ

Beside Q, α and the iteration number, the algorithm for IPIDM uses two other tuning variables denoted by γ and σ which have a pronounced effect on the system dynamic responses. Figure 7 shows the effect σ on ΔF while the other tuning variables are held constant and the controlled system is subjected to a step change of ΔP_d=0.5 %. Table 4 shows the effect of σ with selected suitable values: γ = 50 , Q= 90, *i*=500, α =-1. It illustrates also the system eigenvalues and damping ratio ζ.

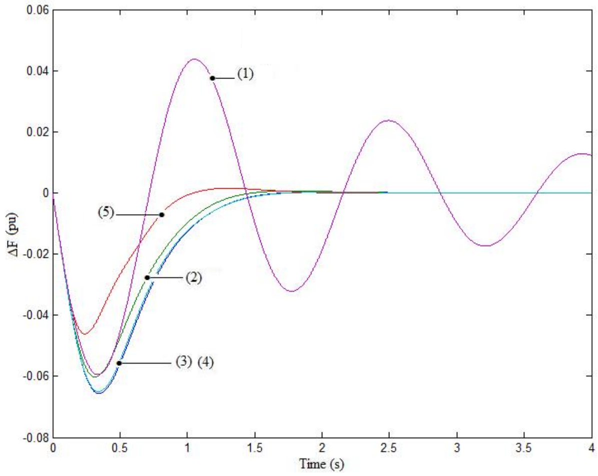


Fig. 7 Effect of different values of σ ($1:\sigma_1$, $2:\sigma_2$, $3:\sigma_3$, $4:\sigma_4$, $5:\sigma_5$)

7.4.2 Effect of γ

The controlled system driven by IPIDM is simulated with different values of γ under the previous disturbance (ΔP_d=0.5 %) while holding other tuning variables constant: σ= *50* , *Q*= *90, i=50*, α = -1. Figure 8 shows this effect on the system dynamic response whereas Table 5 shows the system eigenvalues and damping ratio ζ.

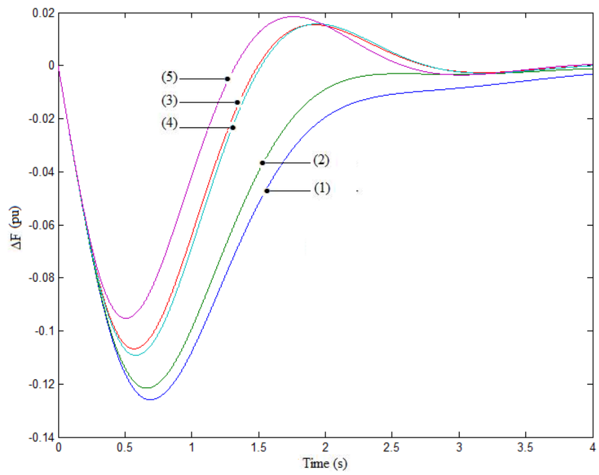


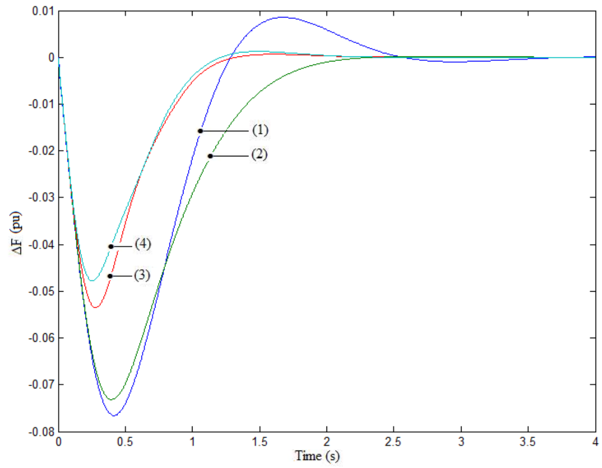
Fig. 8 Effect of different values of γ ($1: \gamma_1$, $2: \gamma_2$, $3: \gamma_3$, $4: \gamma_4$, $5: \gamma_5$)

On the basis of the simulation results, it is clear that the tuning variables have a pronounced influence on the system dynamics. Through proper manipulation of these parameters, an improvement in the control design can be obtained. This represents an advantage which might not exist in other design methods.

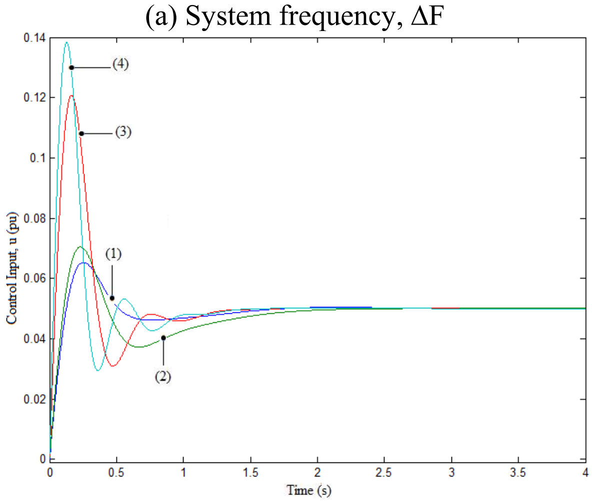
Comparison between the performances of the proposed iterative PID controllers, subjected to several tests, is presented next. The tests in sections 7.5 and 7.7 are done with the system undergoing a load step change of ΔP_d=5%.

7.5 Normal Operating Conditions

The time response of the system frequency ΔF with the system driven by each of the proposed iterative PID controllers, is presented in Fig. 9. It can be noticed that the controllers satisfy the system stability and show acceptable performance (few oscillations and relatively short settling time). It is clearly shown that the IPID controller has large over- and undershoot with a longer settling time. This is expected because it was designed to guaranty the system stability only. The IPIDHI controller shows good performance with a smaller settling time and acceptable undershoot but with a relatively higher effort as shown in the response of its control input response *u*. IPIDH2 shows better response than IPID but not as good as IPIDHI and IPIDM. The last controller, PIDM, shows the best response between the four controllers. The smallest undershoot and no overshoot and short settling time but with the largest effort as shown in *u*.



(a) System frequency, ΔF



(b) Control input, u

Fig. 9 System response following a load step change of ΔP_d=5%
(1:IPID, 2:IPIDH2, 3:IPIDHI, 4:IPIDM)

7.6 Tracking-response

For further comparison study, testing of the effectiveness of the proposed controllers when the system is subjected to tracking of the power demand ΔP_d, as shown in Fig. 10, is carried out. The system responses are shown in Fig. 11. It is clear that IPIDHI and IPIDM show the best performance (no overshoot and short drop and increase in ΔF) as compared to IPID and IPIDH2.

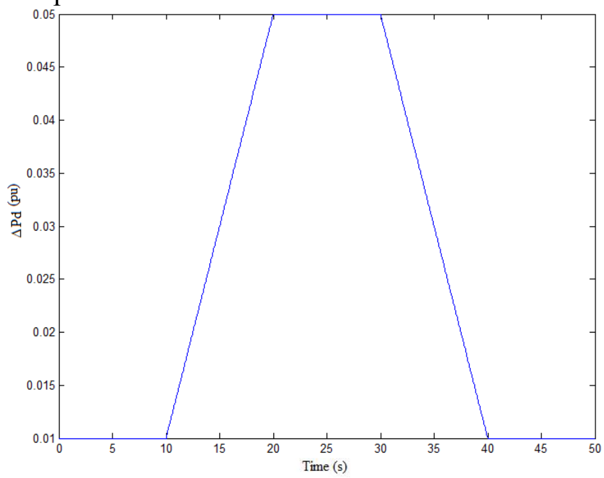


Fig. 10 Tracking of the power demand ΔP_d

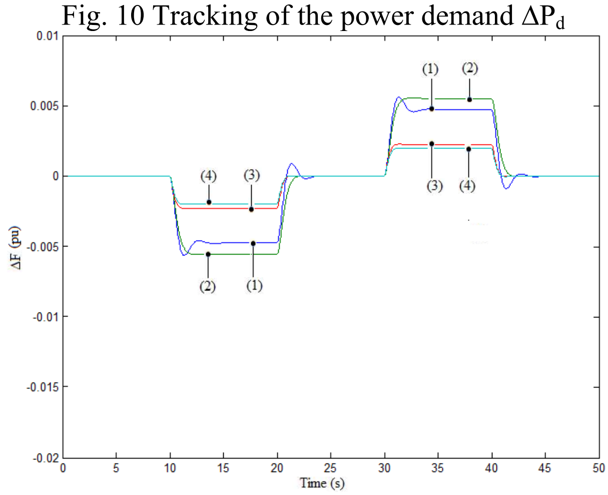
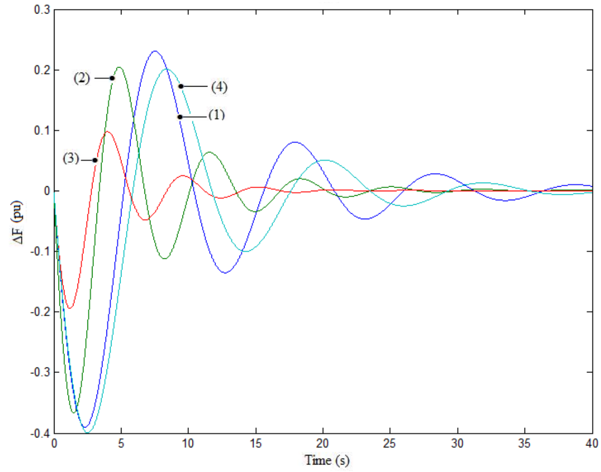


Fig.11 Tracking response of power demand ΔP_d
(1:IPID, 2:IPIDH2, 3:IPIDHI, 4:IPIDM)

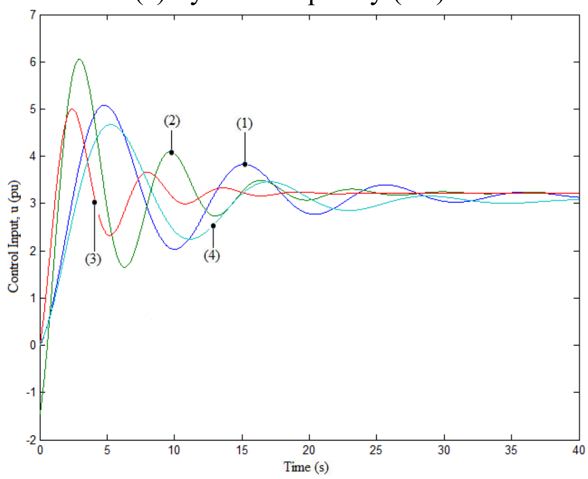
7.7 Parameters Change and Nonlinearity Effect

The system behaviour, including the effect of inherent nonlinearity known as the Generation Rate Constraints (GRC) and a change in the parameters, is presented in the this test and the responses are shown in Fig. 12. GRC represents the physical limitation on the operation of the steam in the power plant. According to [20], it is evaluated as 0.1 pu/min or 0.00167 pu/sec. Parameters (K_p, T_p, T_t, T_g, R) are increased by 50% of their nominal values. It is clear that IPIDM shows the best response whereas IPID exhibits the worse one.

Oscillations are seen with high overshoots in some of the controllers. Fortunately, all controllers overcome both parameters change and system nonlinearity thus verifying the LFC requirements with different levels of robustness.



(a) System frequency (ΔF)



(b) Control input (u)

Fig. 12 Parameters change and Nonlinearity effects (1:IPID, 2:IPIDHI, 3:IPIDM, 4:IPIDH2)

8 Conclusion

This paper has presented a comparison and the design steps for four robust iterative PID controllers. In the first, an Iterative PID that guarantees the system stability (IPID) was presented. In the second and the third ones, iterative PID based on H_2 (IPIDH2) and on H_∞ (IPIDHI) performances, respectively, were investigated. In the last one, an iterative PID designed with maximum output (IPIDM) was presented. All these controllers have their applications in the industrial area.

The proposed designed iterative PID was applied to the Load Frequency Control (LFC) problem of a single-area power system. The effects of the ILMI algorithm variables were investigated and shown. The effectiveness of these controllers was carried out for diverse disturbances and parameters change with the presence of the system inherent nonlinearity (GRC). The results prove that the proposed controllers satisfy the system stability but the robustness is given to PIDM that shows the best responses in all tests but with an important effort being developed by the control input.

References:

[1] K. Ogata, *Modern Control Engineering*, Prentice- Hall Inc., Englewood Cliffs, N.J, U.S.A, 1990.

[2] K.J. Astrom, Theory and Application of Adaptive Control – A Survey, *Automatica*, Vol. 19, No. 5, 1988, pp. 471-486.

[3] K.J. Astrom, T. Hagglund, C.C. Zhang, and W.K. Ho, Automatic tuning and Adaptation for PID controllers –A survey, *IFAC Journal Control Eng. Practice*, Vol. 1, No. 4, 1993, pp. 699-714.

[4] A. M. Abdel Ghany, O.H. Abdalla., I and PID Load-Frequency Controllers for Multi-Area Power Systems, *Proc. 5th Middle East Power System Conference, MEPCON'97*, Alexandria Univ., Egypt, Paper No. 1A-2, January 4-6, 1997, pp. 6-10.

[5] Y.L. Abdel-Magid, M.M. Dawoud, Tuning of AGC of Interconnected Reheat Thermal Systems with Genetic Algorithms, *IEEE Conference Publication on Man and Cybernetics*. Vol. 3, 1995, pp. 2622-2627.

[6] A. Bensenouci, A.M. Abdel Ghany, Optimal Discrete-Time Output Feedback Control for Multi-Area Load Frequency Control Using Evolutionary Programming, *Proc. IEEE International Symposium on Industrial Electronics, ISIE2005,* June 20-23, 2005, Dubrovnik, Croatia.

[7] Y.L. Abdel-Magid, M.A. Abido, AGC tuning of interconnected reheat thermal systems with particle swarm optimization, *Proc. of 10th IEEE Intern. Conference* on *Electronics, Circuits and Systems, ICECS' 2003*, Vol. 1, pp. 376 - 379.

[8] Y.Y. Cao, J. Lam, and Y.X. Sun, Static Output Feedback Stabilization: an ILMI approach, *Automatica*, Vol. 34, 1998, pp. 1641–1645.

[9] F. Zheng, Q.G. Wang, and T.H. Lee, On the design of multivariable PID controllers via LMI approach, *Automatica,* Vol. 38 , 2002, pp. 265–271.

[10] F. Zheng, Q.G. Wang, and T.H. Lee, On the design of multivariable PID controllers via

LMI approach, *Proc. 27th Annual Conference of the IEEE Industrial Electronics Society (IECON'07)*, pp. 776-781.

[11] V.L. Symos, C. Abdallah and P. Dorato, Static Output Feedback: A Survey, *Proc. 33rd Conference on Decision and Control*, Dec. 1994, pp. 837-842.

[12] B. Boivin , L. Rambault , P. Coirault, and C. Dewez, Multivariable output feedback stabilization with structure constraint: application to a gyrometer, *Proc. American Control Conference*, Portland, OR, USA, June 8-10, 2005, pp. 1148-1153.

[13] A.M. Abdel Ghany, Design of Static Output Feedback PID controller via ILMI Method for a Power System Stabilizer, *Proc. 12th Middle East Power Systems Conference MEPCON'2008*, South Valley University, Egypt, March 11-13, 2008, pp. 593-599.

[14] H. Bevrani, Y. Mitani, and K. Tsuji, PI-based multi-objective load-frequency control in a restructured power system, *The IEEE SICE Annual Conference*, Vol. 2, pp. 1745 – 1750, 2004.

[15] H. Bevrani, T. Hiyama, PI/PID based multi-objective control design: an ILMI approach, Proc. Of the IEEE Networking, Sensing and Control, 2005, pp. 750-755.

[16] H. Bevrani, T. Hiyama, Stability and voltage regulation enhancement using an optimal gain vector, *The IEEE Power Engineering Society General Meeting,* 2006.

[17] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in Systems and Control Theory*, SIAM books, Philadelphia, 1994.

[18] Y. Nesterov, and A. Nemirovskii, Interior point polynomial algorithms in convex programming: Theory and applications, *SIAM Studies Appl. Math.*, Vol. 13, 1994.

[19] Gahinet, A. Nemirovski, A. J. Laub, and M. Chiali, *LMI Control Toolbox User's Guide*, MATHWORKS Inc., 1995.

[20] S.C. Tripathy, G.S. Hope, and O.P. Malik, Optimization of Load Frequency Control Parameters with Reheat Steam Turbines and Governor Dead Band Non-Linearity, *The IEE Proc.*, Vol. 129, Pt. C (1), 1982, pp. 10-12.

[21] O. I. Elgerd, *Electric Energy System Theory, An Introduction*, McGraw-Hill, 1971, 1982.

Table 1 IPID Controller for different values of Q and α

No.	Desired values	Obtained values	SOFPID GAINS			Eigenvalues	Damping ratio
	Q	α	F_1	F_2	F_3	λ	ζ
1	0.001	-0.148	-1.055	-3.064	-0.441	-1.24±j2.29 -6.71±j8.23	0.47
2	0.1	-1.143	-1.427	-3.561	-0.484	-6.36±j8.67 -1.58±j2.28	0.57
3	0.12	-1.214	-1.516	-3.913	-0.471	-6.22±j8.35 -1.72±j2.46	0.58
4	1	-1.833	-3.238	-7.408	-0.726	-5.45±j11.3 -2.50±j2.35	0.73
5	5	-1.981	-5.698	-6.711	-1.598	-6.06±j18.9 -1.89±j0.80	0.91

Table 2 IPIDHI design for different values of γ

No	Desired values	Obtained values	SOFPID Gains			Eigenvalues	Damping ratio
	γ	α	F_1	F_2	F_3	λ	ζ
1	0.4	-6.014	-0.139	-0.580	-0.204	-7.13±j4.19 -0.81±j1.21	0.56
2	5	-0.389	-1.665	-0.916	-0.462	-5.50±j8.47 -4.37, -0.52	0.55
3	6	-0.671	-1.697	-1.472	-0.448	-5.38 ±j8.16 -4.21, -0.92	0.57
4	9	-1.052	-2.191	-2.329	-0.570	-5.58±j9.82 -3.38, -1.35	0.49
5	90	- 6.014	-3.472	-7.817	-0.655	-4.76±j10.4 -3.18±2.2	0.83

Table 3 IPIDH2 with different values of γ

No	Desired values	Obtained values	Number of Iteration	SOFPID GAINS			Eigenvalues	Damping ratio
	γ	α	i	F_1	F_2	F_3	λ	ζ
1	15	-0.946	50	-0.997	-0.869	-0.372	-6.09±j7.13 -0.873,-2.828	0.64
2	25	-1.266	70	-1.059	-1.147	-0.369	-5.99±j6.99 -1.30, -2.60	0.65
3	35	-1.349	90	-1.082	-1.273	-0.354	-5.81±j6.59 -1.49, -2.77	0.67
4	60	-1.702	100	-1.242	-1.564	-0.386	-5.80±j7.15 -2.14±0.14	0.98
5	100	-1.588	150	-1.222	-1.657	-0.379	-5.80±j6.99 -2.14±j0.65	0.95

Table 4 IPIDM Controller of different values of γ

No	Desired values	Obtained values	SOFPID Gains			Eigenvalues	Damping ratio
	σ	α	F_1	F_2	F_3	λ	ζ
1	1	-0.389	-27.171	-596.63	-30.938	-0.43±j4.37 -7.52±j87.7	0.09
2	4	-1.022	-6.069	-17.046	-1.069	-4.82±j14.4 -3.13±j2.94	0.73
3	15	-1.276	-2.899	-6.202	-0.587	-3.00±j2.07 -4.94±j9.61	0.82
4	35	-3.581	-2.246	-4.512	-0.4701	-4.82±j7.97 -3.13±j1.80	0.87
5	180	-1.018	-2.299	-4.599	-0.474	-4..76±j8.04 -3.18±j1.74	0.89

Table 5 IPIDM Controller of different values of γ

No.	Desired values	Obtained values	SOFPID GAINS			Eigenvalues	Damping ratio
	γ	α	F_1	F_2	F_3	λ	ζ
1	0.2	-1.76	-0.177	-0.428	-0.076	-1.60±j2.50 -11.64, -1.044	0.54
2	0.3	-1.67	-0.219	-0.544	-0.092	-1.63±j2.46 -11.231, -1.39	0.60
3	0.5	-1.35	-0.397	-1.148	-0.1505	-1.41±j2.49 -9.279, -3.79	0.50
4	0.7	-1.56	-0.352	-1.042	-0.135	-1.36±j2.50 -9.913, -3.25	0.48
5	0.85	-0.411	-0.661	-2.014	-0.254	-1.31±j2.50 -6.63±j4.38	0.47