



**IN THE NAME OF ALLAH,  
MOST MERCIFUL, MOST GRACIOUS**



# **Proceedings of the Seventh Saudi Engineering Conference**

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## **Volume V**

**Research and development to serve the industry and upgrade its services**

Mechanical Engineering

Industrial Engineering



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## **PREFACE**

The Seventh Saudi Engineering Conference comes to complement the series of Saudi engineering conferences which started in 1402H and have been hosted successively by different colleges of engineering of the Saudi universities. The College of Engineering at King Saud University is honored to host the conference for the second time.

These conferences have greatly contributed to the resettlement of technology, the dissemination and exchange of experiences between engineering professionals, and have helped to promote the scientific research besides advancing innovation and excellence.

At a time of the advanced technology and the availability of information in various ways, nations have become closer and the world is turning into a small village, the economy has become the prime engine of the world. It is necessary for all nations to work hard to cope with this technical progress and benefit from it, and moreover create appropriate conditions to deal with this tremendous development and competition as much as possible. It is incumbent upon all professionals in general and engineers in particular to work hard to provide the proper environment in such circumstances.

As a consequence, The Seventh Saudi Engineering Conference discusses an important and vital theme for researchers, engineers and industrialists. The theme is to provide an Engineering Environment to merge in a Competitive Global Economy in an open and boundary-less economy and profession. This conference is trying to answer this question through well-formulated seven topics.

Conference topics discuss multiple issues related to engineering profession and engineering firm, engineering environment through education and labor market requirements, engineering rehabilitation, preservation of the environment, rationalization of resource consumption, Saudi construction code, development of the engineering sector to diversify sources of national income, and research and development to service the industry and upgrade its services.

The conference proceedings contain 168 refereed scientific research papers which are distributed into a number of volumes, and each volume contains one or more topic. A separate volume for paper abstract is also published in addition to electronic proceedings that includes all papers accepted in the conference. These proceedings will be a scientific reference for engineers in the Kingdom and the worldwide.

Finally, thanks to Almighty God for his help in completing of this work and deep thanks for all members of the Conference Committees for their efforts, and special thanks to members of the Scientific Committee for their efforts to have this documentation of the huge scientific research, which is an important reference for researchers and engineers. Thanks also for authors and experts who have contributed their ideas, their research to the success of the conference.

**Thanks**

**Chair of organizing committee**

**Prof. Abdulaziz A. Alhamid**

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## INTRODUCTION

Under the high patronage of his Royal Highness the Prince Sultan Ibn Abdulaziz the crown prince and minister of defense, aviation and inspector general, the College of engineering at the King Saud university hosted the Seventh Saudi Engineering Conference during the period 22 to 25 Dhu Alqeeda 1428 corresponding to 2-5 December 2007. The theme issue of the conference is "Towards An Engineering Environment Competitive to the Economics of Globalization".

The response to contribute in the conference has been most encouraging. A large number of abstracts were received. After a thorough peer-review process for evaluating the submitted papers, the scientific committee has selected a total of 168 papers, presented by 300 researchers. The conference has drawn participants from the different kingdom universities, colleges, institutes and technical education establishments as well as governmental and national companies. The conference has also attracted international participation from universities and institutes of United Arab Emirates, Egypt, Sudan, Algeria, Tunisia, Malaysia, India, Great Britain, Germany, France, Deutschland, Canada, Japan and United States of America.

One of the main objectives of the conference was to contribute to the review and development of important aspects of the engineering sector both public and private. The topics of the conference were chosen to tackle the challenges that engineering education and its outputs are facing. In addition, the themes also emphasized on the contribution of the engineers to the development of the country. The conference themes were as follows:

- Engineering qualification and its role in the strategy of Saudization
- Engineering specialties as viewed from the educational establishments and the job market requirements
- Engineering sector contribution to resources conservation
- Engineering and environmental protection

- The Saudi building code
- Development of the engineering sector for diversification of income resources
- Research and development in the service of industry and for the improvement of services

In addition to the specialized scientific papers that covered the above mentioned themes the conference also hosted a number of plenary lectures and discussion forums that attracted the participation of key policy makers as well as academics and economic parties.

The selected abstracts and papers have been documented in the proceedings which comprise of six volumes in accordance with the conference themes. The papers are also documented in CDs.

Before concluding I would like to express my gratitude to all members of the Scientific Committee for their efforts and active participation to the success of the conference. Thanks are also due to the referees who have been of great help in selecting high quality papers for the conference. The support provided by the secretarial and technical staff of the college of engineering is also thankfully acknowledged.

Finally on my own behalf and behalf of the Scientific Committee I would like to record our appreciation and sincere thanks to His Excellency the rector of the King Saud University and the Dean of College of engineering, the chairman of the organizing committee for their continued support and valuable guidance, We are all hopeful that this scientific conference will be of a support for recruiting engineering specialties on a larger scale and contribute to the growth and prosperity of our country. May Allah Almighty accept our sincere efforts.

**Chairman of the Scientific Committee**

**Prof. Khalid Ibrahim Alhumaizi**

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## ON RELIABILITY-COST CHARACTERIZATION OF A PARTIALLY REDUNDANT SYSTEM

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### ABSTRACT

Simple reliability-cost metrics (such as reliability per cost or life expectancy per cost) can be used to guide the selection of a system from among several systems that are candidates for providing the same performance in a given mission. Other more elaborate metrics (such as the cost elasticity of reliability  $\epsilon_{R,C}$  or cost elasticity of life expectancy  $\epsilon_{T,C}$ ) can be used to assess the cost-benefit aspect of adding redundancy to a system with the purpose of enhancing its reliability. We study the  $\epsilon_{R,C}$  and  $\epsilon_{T,C}$  metrics analytically and numerically for the partially redundant or  $k$ -out-of- $n$ :G(F) system (or  $k$ -out-of- $n$  system, for short). We believe the  $\epsilon_{T,C}$  metric is a more tangible and a more cumulative measure than the  $\epsilon_{R,C}$  metric. The expression of  $\epsilon_{R,C}$  is dependent on component reliability or failure rate, and is highly susceptible to round-off errors to the extent that catastrophic cancellations take place. By contrast, our expression for  $\epsilon_{T,C}$  is independent of component characteristics and is really insensitive to round-off errors since it is a purely additive formula. We provide charts for  $\epsilon_{T,C}$  that a reliability engineer can use to assess the cost incurred in achieving a certain life expectancy for a partially redundant system. These charts are also applicable to any coherent system, since the life expectancy for a coherent system can be approximated by that of a partially redundant system.

### KEY WORDS

Cost, Reliability, Life expectancy (Mean time to failure), Partially redundant or  $k$ -out-of- $n$ :G(F) system.

## I. INTRODUCTION

The reliability  $R(t)$  of a system is the probability that the system will adequately perform its specified purpose for a specified period of time  $(0, t]$  under specified environmental conditions<sup>[1]</sup>. Suppose that a certain engineering job or function can be equally done or performed over the time period  $(0, t]$  by several candidate systems. If system  $i$  has reliability  $R_i(t)$  and cost  $C_i$ , then classical reliability-cost considerations makes it imperative to choose the system that has the maximum reliability per cost, i.e., the one that satisfies

$$\underset{i}{\text{maximum}} \quad R_i(t)/C_i. \quad (1)$$

If the time span for operating the system is not known in advance, then a more prudent choice can be based not on the reliability of the system but rather on its life expectancy or Mean Time To Failure (MTTF)  $T_i$ . Hence, the preferred candidate system is the one having the maximum life expectancy per cost, i.e., the one satisfying

$$\underset{i}{\text{maximum}} \quad T_i/C_i. \quad (2)$$

Beside the simple metrics in (1) and (2), which guide the selection of a system from among several candidate systems of equivalent performance, there are more elaborate metrics that can be used to assess the cost-benefit aspect of adding redundancy to a system with the purpose of enhancing its reliability. Notable among these are two metrics: (a) the cost elasticity of reliability  $\epsilon_{R,C}$  [2, 3], and (b) the cost elasticity of life expectancy or MTTF [3]. We study the  $\epsilon_{R,C}$  and  $\epsilon_{T,C}$  metrics analytically and numerically for the partially redundant or k-out-of-n:G(F) system (or k-out-of-n system, for short), which is a system of  $n$  components that functions (fails) if at least  $k$  out of its  $n$  components function (fail). Situations in which this system serves as a useful model are frequently encountered in practice [4]. The k-out-of-n system plays a central role for the general class of coherent systems, as it can be used to approximate the reliability of such systems [5]. While virtually all nontrivial network reliability problems are known to be NP-hard for general networks [6], the regular structure of the k-out-of-n system allows the existence of efficient algorithms for its reliability analysis that are of quadratic-time linear-space complexity in the worst case [4, 7-10]. The k-out-of-n:G system covers many interesting systems as special cases. These include the perfectly reliable system ( $k=0$ ), the parallel system ( $k=1$ ), the voting or N-modular redundancy (NMR) system ( $k = \lceil (n+1)/2 \rceil$ ), the fail-safe system ( $k=n-1$ ), the series

system ( $k=n$ ), and the totally unreliable system ( $k=n+1$ ). We use the terms  $k$ -out-of- $n$  system and partially redundant system as synonymous, though some authors [11, 12] restrict their equivalence to the case  $1 < k < n$ , which lies somewhere between the extreme cases of the (non-redundant) series system and the (fully-redundant) parallel system. The  $k$ -out-of- $n$ :G system and the  $k$ -out-of- $n$ :F system are mirror images of each other; their successes are dual switching functions. The  $k$ -out-of- $n$ :G system is exactly equivalent to the  $(n-k+1)$ -out-of- $n$ :F system[4].

While redundancy in a parallel system is always useful, it is not so for a  $k$ -out-of- $n$ :G system with  $k > 1$ . For this latter system, redundancy does not improve system reliability unless the basic component reliability is higher than a certain threshold value [13]. Since redundancy is not achieved without an increase in cost, one should make sure that it is useful.

The remainder of this paper is organized as follows. Section II reviews and compares methods for computing the reliability and life expectancy of a  $k$ -out-of- $n$  system. Section III derives an expression and reports computational results for  $\epsilon_{R,C}$ , while section IV does the same for  $\epsilon_{T,C}$ . Section V concludes the paper.

## II. RELIABILITY AND LIFE EXPECTANCY FOR A $k$ -out-of- $n$ SYSTEM

The reliability of a  $k$ -out-of- $n$ :G system (with independent components of identical reliabilities  $R_0$ ) is given by [3, 4, 14, 15]:

$$R = \sum_{m=k}^n c(m, n) R_0^m (1 - R_0)^{n-m}, \quad (3)$$

$$= \sum_{m=k}^n (-1)^{m-k} c(k-1, m-1) c(m, n) R_0^m, \quad (4)$$

where  $c(m, n)$  is the combinatorial or binomial coefficient ( $n$  choose  $m$ ). Formula (4) is considered more suitable than formula (3) for hand calculation [14, 15], because (4) expresses  $R$  as a polynomial of  $R_0$  only, while (3) involves powers of  $R_0$  and  $(1 - R_0)$ . The life expectancy or Mean Time To Failure (MTTF) of a general non-repairable or mission-type system is given by

$$T = MTTF = \int_0^{\infty} R(t) dt. \quad (5)$$

For a k-out-of-n:G system having components subject to a common constant failure rate (CFR)  $\lambda$ , the component reliability is

$$R_0(t) = e^{-\lambda t}, \quad t \geq 0, \quad (6)$$

and the *MTTF* of the system is obtained from equations (4) – (6) as

$$\begin{aligned} T &= \int_0^{\infty} \sum_{m=k}^n (-1)^{m-k} c(k-1, m-1) c(m, n) (e^{-\lambda t})^m dt, \\ &= \sum_{m=k}^n \frac{(-1)^{m-k}}{\lambda m} c(k-1, m-1) c(m, n). \end{aligned} \quad (7)$$

A simpler expression for T, however, can be obtained from the state diagram for a k-out-of-n:G system [3], namely

$$T = \frac{1}{\lambda} \sum_{m=k}^n \frac{1}{m}. \quad (8)$$

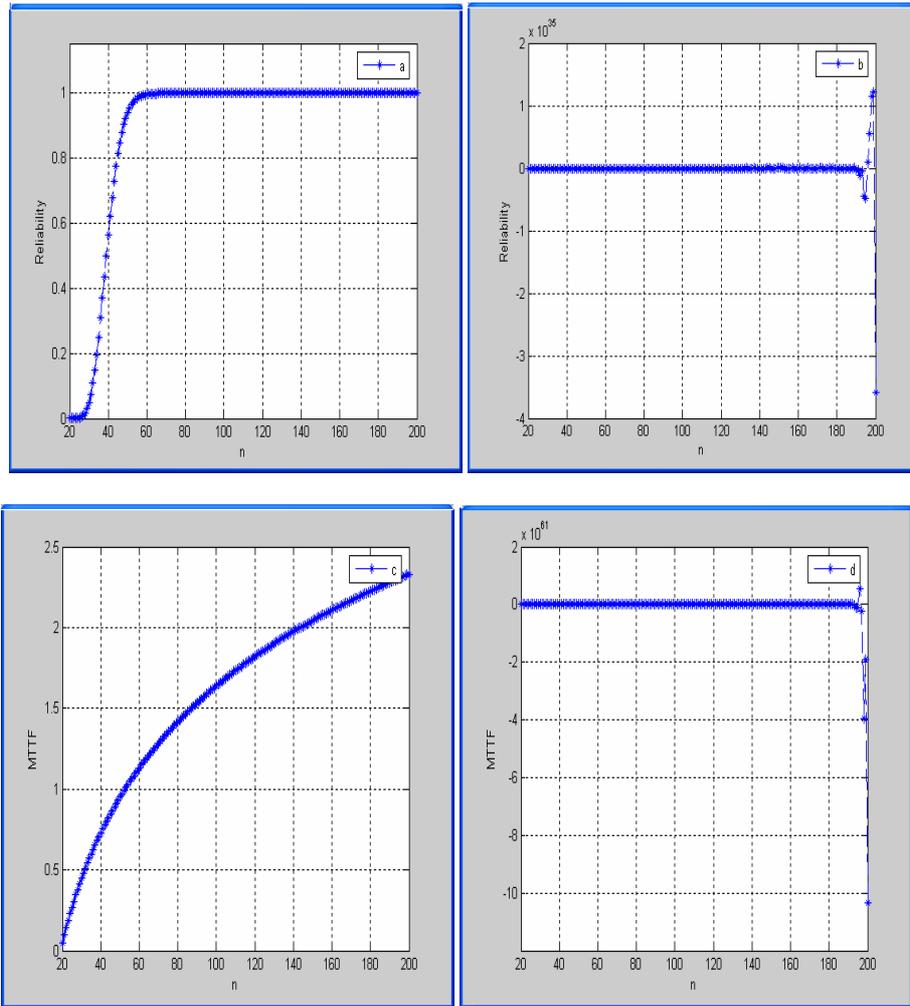


Fig. 1. Reliability and life expectancy versus  $n$  for a 20-out-of- $n$ :G system

Figure 1 demonstrates our computational experience with the above formulas. It presents the reliability and life expectancy versus  $n$  for a 20-out-of- $n$ :G system. In Figs. 1(a) and 1(b), the reliability  $R$  is computed via formulas (3) and (4), respectively, while in Figs. 1(c) and 1(d), the life expectancy is computed via formulas (8) and (7), respectively. Despite the fact that formulas (4) and (7) are frequently cited in the reliability literature, they are, in fact, worthless from the numerical point of view. These two formulas are highly susceptible to round-off errors and severely suffer from catastrophic cancellations, and therefore they produce the highly erratic plots in Figs. 1(b) and 1(d). In retrospect, the undesirable behavior of formulas (4) and (7) should have been anticipated since they are outcomes of the inclusion-exclusion technique<sup>[4]</sup>. By contrast, formulas (3) and (8) are purely additive formulas and are really insensitive to round-off errors. We will base our further work herein on these two formulas.

### III. COST ELASTICITY OF RELIABILITY

The cost elasticity of reliability is defined as<sup>[2]</sup>

$$\epsilon_{R,C} = (\Delta R / R) / (\Delta C / C). \quad (9)$$

This metric measures the relative change in reliability  $R$  that can be obtained for a given relative change in cost  $C$ . As its name indicates, this metric mimics a well-known material constant, viz., the modulus of elasticity which relates an applied stress to the resulting strain or relative change in length<sup>[16]</sup>. However, the metric  $\epsilon_{R,C}$  is more analogous, from a cause-effect point of view, to the price elasticity of demand or supply, a concept well known in microeconomics<sup>[17]</sup>. For a  $k$ -out-of- $n$ :G system, if we let the number of components  $n$  change to  $(n + \Delta n)$ , then the reliability  $R$  in (3) changes to  $(R + \Delta R)$  given by

$$R + \Delta R = \sum_{m=k}^{n+\Delta n} \left( c(m, n + \Delta n) R_0^m (1 - R_0)^{n+\Delta n-m} \right). \quad (10)$$

From (3) and (10), we can express the change  $\Delta R$  in reliability due to a unit change  $\Delta n=1$  in the number of components as

$$(\Delta R)_{\Delta n=1} = [c(n+1, n+1) R_0^{n+1} + \sum_{m=k}^n (c(m, n+1) R_0^m (1 - R_0)^{n+1-m} - c(m, n) R_0^m (1 - R_0)^{n-m})]. \quad (11)$$

Using the binomial identifies

$$c(n+1, n+1) = 1, \tag{12}$$

$$c(m, n+1) - c(m, n) = c(m-1, n), \tag{13}$$

we reduce (11) to

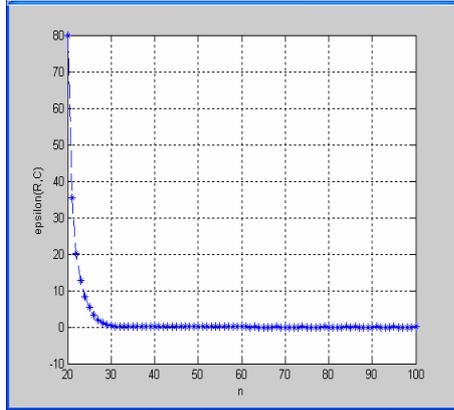
$$(\Delta R)_{\Delta n=1} = R_0^{n+1} + \sum_{m=k}^n R_0^m (1-R_0)^{n-m} [c(m-1, n) - c(m, n+1)R_0], \tag{14}$$

and finally, obtain the cost elasticity of reliability as

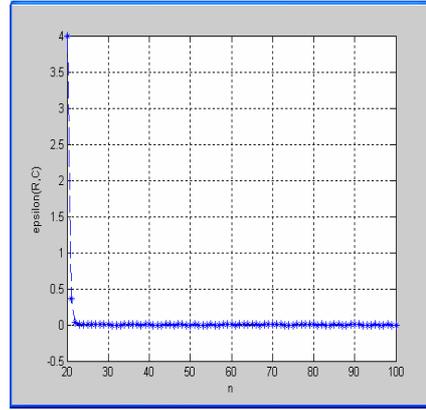
$$\epsilon_{R,C} = \frac{n}{R} \left[ R_0^{n+1} + \sum_{m=k}^n R_0^m (1-R_0)^{n-m} c(m-1, n) \left(1 - \frac{n+1}{m} R_0\right) \right]. \tag{15}$$

Though formula (3) is a purely additive formula that expresses  $R$  as the sum of nonnegative terms, formula (15) is not an additive formula unless  $1 > ((n+1)/k)R_0$ , i.e., unless  $R_0 < k/(n+1)$ .

Figures 2 and 3 present a cost-reliability characterization ( $\epsilon_{R,C}$  versus  $n$ ) for a 20-out-of- $n$ :G system with component reliabilities  $R_0 = 0.8$  and  $R_0 = 0.99$ , respectively. Formula (15) for the cost elasticity of reliability  $\epsilon_{R,C}$  gives satisfactory results up to  $R_0 = 0.8$  (Fig. 2) and then starts to exhibit some unacceptable negative values (values of -0 rather than +0), i.e. it exhibits erratic behavior for very small or negligible values of  $\epsilon_{R,C}$  (Fig. 3). We must stress that the erratic behavior obtained is solely due to aggravated cumulative round-off error and is definitely not a result of some error in formulation or programming. Formula (15) gives acceptable and verifiable results for a wide range of values of  $k$ ,  $n$ , and  $R_0$ . However, it fails to assess  $\epsilon_{R,C}$  properly for systems having good components (i.e., for systems of practical interest). Anyhow, for such systems  $\epsilon_{R,C}$  diminishes and becomes indistinguishable from zero.



**Fig. 2. Cost-reliability characterization of a 20-out-of- $n$ :G system with component reliabilities  $R_0 = 0.8$  .**



**Fig. 3. Cost-reliability characterization of a 20-out-of- $n$ :G system with component reliabilities  $R_0 = 0.99$  .**

#### IV. COST ELASTICITY OF LIFE EXPECTANCY

From cost considerations, life expectancy seems to be a more tangible and cumulative measure than reliability itself. Therefore, we introduced in an earlier paper <sup>[3]</sup> the concept of the cost elasticity of life expectancy or *MTTF*, which we defined as

$$\epsilon_{r,c} = \frac{(\Delta T / T)}{(\Delta C / C)} = \frac{(\Delta T / T)}{(\Delta n / n)}, \quad (16)$$

For a  $k$ -out-of- $n$ :G system, if we let the number of components  $n$  change to  $(n + \Delta n)$  in (8), then the Life Expectancy changes to  $(T + \Delta T)$  given by

$$T + \Delta T = \frac{1}{\lambda} \sum_{m=k}^{n+\Delta n} \frac{1}{m}. \quad (17)$$

From (8) and (17), we can express the change  $\Delta T$  in  $T$  due to a unit change  $\Delta n = 1$  in the number of components as

$$(\Delta T)_{\Delta n=1} = \frac{1}{\lambda} \frac{1}{n+1}, \tag{18}$$

and hence, we can express  $\epsilon_{T,C}$  as

$$\epsilon_{T,C} = \frac{\frac{1}{\lambda} \frac{1}{n+1} n}{\frac{1}{\lambda} \sum_{m=k}^n \frac{1}{m}} = \frac{\frac{n}{n+1}}{\sum_{m=k}^n \frac{1}{m}} = \frac{\frac{n}{n+1}}{\frac{1}{k} + \frac{1}{k+1} + \dots + \frac{1}{n-1} + \frac{1}{n}}. \tag{19}$$

The cost elasticity  $\epsilon_{T,C}$  of the life expectancy of a  $k$ -out-of- $n$ :G system is a function of  $n$  and  $k$  only and is independent of the component reliability  $R_0$  and the component failure rate  $\lambda$ .

Noting that the sum  $S = \sum_{m=k}^n \frac{1}{m}$  satisfies the following inequalities for  $k > 1$

$$S > \int_k^{n+1} \frac{dx}{x} = \ln\left(\frac{n+1}{k}\right), \tag{20}$$

$$S < \int_k^{n+1} \frac{dx}{x-1} = \ln\left(\frac{n}{k-1}\right), \tag{21}$$

we obtain the following tight bounds on  $\epsilon_{T,C}$

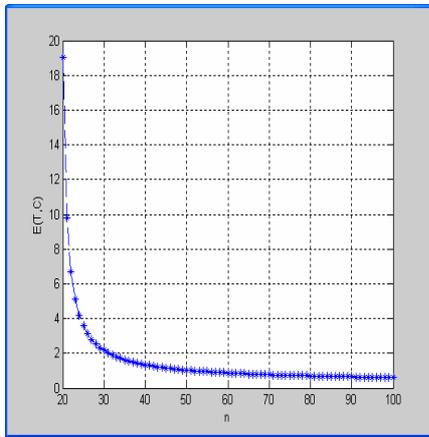
$$\frac{\frac{n/(n+1)}{\ln\left(\frac{n}{k-1}\right)}} < \epsilon_{T,C} < \frac{\frac{n/(n+1)}{\ln\left(\frac{n+1}{k}\right)}}. \tag{22}$$

Table 1 lists  $\epsilon_{T,C}$  values in proper-fraction form (exact integer arithmetic) for small  $k$  and  $n$  values. Figure 4 represents the cost elasticity of life expectancy for a 20-out-of- $n$ :G system versus the number of components  $n$ . Figure 5 represents the cost elasticity of life expectancy for a  $k$ -out-of-50:G system versus the number of components  $k$  required for system success. Table 1 and Figure 5 explain why the

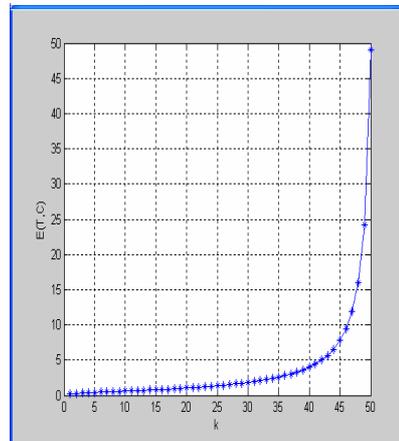
fail-safe  $((n-1)$ -out-of- $n$ :G system or 2-out-of- $n$ : F system) is so popular. Among redundancy systems it has the best cost for added redundancy since it has the highest  $\epsilon_{T,C}$ . Of course, the series system ( $n$ -out-of- $n$ :G system or 1-out-of- $n$ :F system) has an  $\epsilon_{T,C} = (n^2 / (n+1))$  that is higher than that of the fail-safe system, but the series system has no redundancy at all and cannot tolerate even a single failure.

**Table 1. Value of  $\epsilon_{T,C}$  as a function of  $k$  and  $n$  for  $1 \leq k \leq n \leq 6$ .**

$n \backslash k$	1	2	3	4	5	6
1	0.5000000	0.44444444	0.90909091	0.38400000	0.36496350	0.34985423
2		1.33333333	0.90000000	0.73846154	0.64935065	0.59113300
3			2.25000000	1.37142857	1.06382979	0.90225564
4				3.20000000	1.85185185	1.38996139
5					4.16666667	2.33766234
6						5.14285714



**Fig. 4. Cost elasticity of life expectancy for a 20-out-of- $n$ :G system versus its number of components  $n$ .**



**Fig. 5. Cost elasticity of life expectancy for a  $k$ -out-of-50:G system versus the number of components  $k$**

**required for system success.**

## V. CONCLUSION

An important goal for reliability engineering is to achieve cost minimization. However, this goal has rarely been achieved, primarily because of the lack of suitable mathematical models or metrics [3]. Two recently introduced metrics that explicitly deal with cost in reliability engineering are the cost elasticity of reliability  $\in_{R,C}$  and the cost elasticity of life expectancy  $\in_{T,C}$ . These two metrics were studied herein, both analytically and numerically, for a  $k$ -out-of- $n$  system.

The  $\in_{T,C}$  metric has several advantages when compared with the competitive  $\in_{R,C}$  metric. One advantage stems from the fact that the  $MTTF$  is a cumulative, integral, or averaging measure for reliability itself. For the wide class of  $k$ -out-of- $n$ :G(F) systems  $\in_{T,C}$  depends only on  $k$  and  $n$  while  $\in_{R,C}$  depends on component characteristics in addition to its dependence on  $k$  and  $n$ . For such systems, it was possible to express  $\in_{T,C}$  by a purely additive formula that is insensitive to round-off errors, while the  $\in_{R,C}$  formula is very susceptible to round-off errors to the extent that catastrophic cancellations take place. Moreover, the  $\in_{T,C}$  metric decreases but remains distinguishable from zero for large  $n$ , while the  $\in_{R,C}$  metric diminishes and becomes indistinguishable from zero for large  $n$  and  $R_0$ . This fact imposes a limitation on the utility of both metrics for large ultra-reliable systems. A detailed comparison between the aforementioned two metrics is given by Alsolami [18].

Though the main theme of this paper is the study of reliability-cost metrics, it has a very important offshoot. The paper demonstrates that certain celebrated formulas that are widely used in the reliability circles are computationally worthless, as they severely suffer from catastrophic cancellation.

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